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THE ART OF SOLVING PROBLEMS IN HIGHER ARITHMETIC.

REV. J. HUNTER, M.A.

PROBLEMS IN HIGHER ARITHMETIC

LONDON : PRINTED BY
SPOTTISWOODE AND CO., NEW STREET SQUARE
AND PARLIAMENT STREET

THE ART OF SOLVING

PROBLEMS IN HIGHER ARITHMETIC

WITH A CHAPTER ON THE ARITHMETIC OF LATITUDE AND LONGITUDE

A TRAINING BOOK FOR SENIOR SCHOOLBOYS AND FOR EXAMINATION CANDIDATES

BY THE

REV. J. HUNTER, M.A.

LONDON
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1884

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PREFACE.

It is to be regretted that the practice of what is called *coaching* and *cramming* should ever have to be resorted to in Arithmetic—a science which, in the deliberate study of it, involves a discipline so conducive to the proper growth of intellect.

In the pursuit of any kind of useful knowledge it has often been found that they who, by themselves, have patiently and perseveringly grappled with difficulties have attained the best success. And, certainly, teaching, which in former times gave too little assistance, ought now to leave more in the hands of independent effort than it commonly does.

There is, however, one portion of Arithmetic in which, more than in any other, a learner requires help, if it can be given with reference to some broad, general principle; it is the department of Miscellaneous Problems in Higher Arithmetic, when no particular Rule seems to be indicated as sufficient for solution. Help is here wanted, we say, if it can be given with reference to some broad, general principle, not merely augmenting knowledge, but impregnating the mind itself to make it grow; and it has appeared to the

author of the present publication, that such miscellaneous problems are to a considerable extent reducible to classes, in each of which there is some common characteristic, which, when successively studied and understood, will aid much in the cultivation of general skill. To assist a pupil in the solution of first one problem and then another of a different kind can do little towards good progress; but to confine his attention at one time to a series of similar examples, extracted from a miscellaneous collection, will most effectually train him into an aptitude for the general work of Higher Arithmetic.

After all the aid that may be derived from this manual, it is intended and desirable that it should contain difficulties to be encountered; but in order that time may be duly economised, and opportunity given to the student of estimating his own work, a Key will shortly be published, for such legitimate use as will be readily inferred.

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THE ART OF SOLVING

PROBLEMS IN HIGHER ARITHMETIC.

INTRODUCTION.

1. THE more difficult kind of Miscellaneous Questions, such as are commonly proposed at the end of a Treatise on Arithmetic, consists of problems not so stated as to admit the direct application of any of the regular Rules, but making known some quantity that usually forms the answer to given conditions in a direct elementary example, and suppressing some quantity that usually forms a given condition.

2. Readiness in discerning how such problems are to be solved may be much promoted by the practice of constructing and working examples that invert in various ways the usual order recognised in an elementary Rule.

Take for illustration an example in simple interest, requiring to find the amount of £325 in 3 years at $4\frac{1}{2}$ per cent. per annum. The best way of considering this question is by observing that as the interest on 100 is to be $4\frac{1}{2} \times 3$, or $13\frac{1}{2}$, the amount of 100 will be $113\frac{1}{2}$,

and, therefore, as often as the given principal £325 contains the principal £100, so often will the required amount contain the amount of £100; that is, the required amount is equal to $\frac{113\frac{1}{2}}{100}$ of £325 = $\frac{227}{200}$ of £325 = $\frac{227}{800}$ of £1300 = $\frac{1}{8}$ of £2951 = £368$\frac{7}{8}$.

Now, let the question be inverted as follows:—

Find the principal that will amount to £368 17*s.* 6*d. by simple interest at* 4$\frac{1}{2}$ *per cent. in* 3 *years.*

The consideration now should be, that as often as the proposed amount £368$\frac{7}{8}$ contains the amount of £100, so often will the required principal contain the principal £100; that is, the required principal is equal to $\frac{368\frac{7}{8}}{113\frac{1}{2}}$ times £100 = $\frac{2951}{908}$ of £100 = £325. *Ans.*

Thus the more direct and concise the views we take of the originating question, the easier is it to see how the derived and inverted question may be dealt with.

3. When an originating example consists of a series of operations, some of which are additions or subtractions of absolute, independent, quantities, a transformed statement of the example may in some instances be easily worked by Arithmetic.

The following data will suggest a specimen of what is here referred to:—

A man possessing £270 receives an addition of £60, and pays away a fifth of the amount; he then receives £96, and pays away an eighth of all he has.

Here the successive steps are as follows:—After the first payment there remains $\frac{4}{5}$ of £330 = £264, which is increased to £360; then the second remainder is

equal to $\frac{7}{8}$ of £360 = £315, which is seen to be one-sixth more than the owner had at first. These steps serve to construct the following problem:—

A man possesses a certain sum of money, and on receiving £60 more pays away a fifth of the amount; he then receives £96, but after that pays away an eighth of his money, and he has now one-sixth as much again as he had at first. Find the original money.

Calling the original money M, this question may be solved arithmetically as follows:—

On his receipt of the £96 he had $\frac{4}{5}$ of M + $\frac{4}{5}$ of £60 + £96 = $\frac{4}{5}$ of M + £144; then spending $\frac{1}{8}$ of that amount, he has remaining $\frac{7}{8}$ of it, which, by the question, is equal to $\frac{7}{6}$ of M; so that M is equal to $\frac{7}{8} \div \frac{7}{6}$, or $\frac{3}{4}$ of the last amount; that is, equal to $\frac{3}{4}$ of ($\frac{4}{5}$ of M + £144), or equal to $\frac{3}{5}$ of M + £108; hence $\frac{2}{5}$ of M is equal to £108, and therefore M = $\frac{5}{2}$ of £108 = £270. *Ans.*

This, however, is very clumsy procedure; the answer would have been more conveniently found by Algebra. The only advantage it has is its being descriptive and explanatory throughout, for there is no such thing as x pounds in an algebraic equation; x may denote *a number* of pounds; it is always an abstract quantity. All problems that yield simple equations in Algebra may be worked by mere Arithmetic, though in most instances less easily, and very often not so neatly.

4. But when the quantities in a problem, though indirectly and intricately presented, have a *factorial* dependence on one another; that is, when they are related to one another by multiplication or division, Arithmetic can often exhibit the best mode of solution.

The following problem—not a very hard one—will exemplify this:—

I bought a cargo of oranges at the rate of two a penny, and selected $\frac{3}{7}$ *of them for shipment to Antwerp in cases of* 456, *at* 34s. 10d. *per case. Of the remainder I sold* $\frac{10}{19}$ *at* 9d. *a dozen; and of those still remaining I sold* $\frac{7}{9}$ *at the rate of* 20 *oranges for a shilling. There were now left* 320, *but in damaged condition, and I accepted an offer of* 2s. 4d. *for the lot. What was my profit per cent. on the whole? and of how many oranges did the cargo consist?*

Here all the quantities are linked together by ratio, and we may therefore represent the cargo by unity. First, then, finding the 34*s.* 10*d.* received for 456 oranges to be at the rate of 418*d.* ÷ 456 or $\frac{11}{12}d.$ per orange, and noting that 320 sold for 28*d.* is at the rate of $\frac{7}{80}d.$ for each, we may proceed as follows:—

$$\begin{aligned}
\tfrac{3}{7} \text{ at } \tfrac{11}{12}d. &= \tfrac{11}{28}d. \\
\tfrac{10}{19} \text{ of } \tfrac{4}{7} \text{ at } \tfrac{3}{4}d. &= \tfrac{30}{133}d. \\
\tfrac{7}{9} \text{ of } \tfrac{9}{19} \text{ of } \tfrac{4}{7} \text{ at } \tfrac{3}{5}d. &= \tfrac{12}{95}d. \\
1 - (\tfrac{3}{7} + \tfrac{40}{133} + \tfrac{4}{19}) = \tfrac{8}{133} \text{ at } \tfrac{7}{80}d. &= \tfrac{1}{190}d. \\
\text{sum} &= \tfrac{3}{4}d.
\end{aligned}$$

*** Notice a convenient way of obtaining the sum of these proportionate amounts:—

$\frac{1}{190} + \frac{12}{95} = \frac{5}{38}$; then $\frac{35}{266} + \frac{60}{266} = \frac{5}{14}$; then $\frac{10}{28} + \frac{11}{28} = \frac{3}{4}$.

Thus the proportionate return for the outlay of 1 at $\frac{1}{2}d.$ is found to be $\frac{3}{4}d.$, showing a profit of 50 per cent. 1*st Ans.*, and as the 320 oranges were the remaining $\frac{8}{133}$ of the cargo, the whole cargo consisted of 5320 oranges, 2*nd Ans.*

5. Certain variations of which proportion statements are capable deserve the student's attention. We will explain one of the most useful of these.

Take the proportion 133 : 95 :: 42 : 30;
this means that the ratio $\frac{133}{95}$ is identical with the ratio $\frac{42}{30}$, each form of expression being equal to $\frac{7}{5}$.

Now, one of the variations that may be made of the above statement is that which substitutes for the first and third terms, or for the second and fourth terms, the differences of the terms of each ratio; thus,

$$133 - 95 : 95 :: 42 - 30 : 30,$$
$$\text{and } 133 : 133 - 95 :: 42 : 42 - 30.$$

The former of these statements asserts that $\frac{38}{95}$ equals $\frac{12}{30}$; the latter „ „ that $\frac{133}{38}$ equals $\frac{42}{12}$. And why all this should be so is easily made evident; for if $\frac{133}{95} = \frac{42}{30}$, then $\frac{133}{95} - 1 = \frac{42}{30} - 1$, or $\frac{38}{95} = \frac{12}{30}$, and if $\frac{38}{95} = \frac{12}{30}$, then $\frac{95}{38} = \frac{30}{12}$, and therefore $\frac{95}{38} + 1 = \frac{30}{12} + 1$, or $\frac{133}{38} = \frac{42}{12}$.

We will now exemplify with a few problems what has just been taught.

The numerator and denominator of the fraction $\frac{53}{71}$ are to be equally increased, so that the value of the resulting fraction shall be $\frac{7}{9}$. What number must be added to each term?

The denominator of the resulting fraction will, of course, be to its numerator as 9 : 7; therefore, as 9 − 7 : 7 :: the difference of the terms of the resulting fraction (= 71 − 53) : the numerator of the resulting fraction; hence,

$$9 - 7 : 7 :: 71 - 53 : 63, \text{ the new numerator;}$$

so that the number sought = 63 − 53 = 10. *Ans.*

The denominator of a vulgar fraction exceeds its numerator by 226, *and the three-place decimal approximating nearest to the value of the fraction is* ·615. *Find the fraction.*

The fraction is to be very nearly $= \frac{615}{1000}$ or $\frac{123}{200}$; therefore, 200 − 123 : 123 :: 226 : the required numerator $= \frac{226 \times 123}{77} = 361{\cdot}013$; hence the required fraction is $\frac{361}{587}$. *Ans.*

A mixture is made of 43 *quarts brandy and* 14 *quarts water. How much brandy must be added to it, that* 17 *quarts of the mixture may contain a gallon of water?*

At first $\frac{43}{57}$ of the mixture is brandy, and by the required addition the strength is to become equal to $\frac{13}{17}$, and will be expressed by a fraction of which the denominator will exceed the numerator by 57 − 43 or 14; therefore, 17 − 13 : 13 :: 57 − 43 : $45\frac{1}{2}$, the numerator; hence $45\frac{1}{2} - 43 = 2\frac{1}{2}$ quarts brandy. *Ans.*

For thus the brandy will be $\frac{45\frac{1}{2}}{59\frac{1}{2}} = \frac{91}{119} = \frac{13}{17}$ of the mixture, as required.

CHAPTER I.

THE SUM AND THE DIFFERENCE OF TWO QUANTITIES BEING GIVEN: TO FIND THE QUANTITIES.

1. WHEN the sum, and also the difference, of two quantities is known, *adding* the difference to the sum will give twice the greater quantity; *subtracting* the difference from the sum will give twice the less quantity.

This at once appears from the algebraic identities,

$$(x+y)+(x-y)=2x, \text{ and } (x+y)-(x-y)=2y.$$

But it is simply enough understood by considering that if we add to the sum of two quantities what the less wants to make it equal to the greater, we shall have double the greater; or if we take away from the sum of two quantities that by which the greater exceeds the less, we shall leave double the less quantity. If the sum 20 shillings is to be divided between Richard and Henry, so that Richard's share may be 3 shillings more than Henry's, place the sum on a table, and let Richard take 3 shillings from it; then he will have 3 shillings more than Henry if what remains, viz. 17 shillings, be divided equally between them; thus Henry will get 8*s*. 6*d*. and Richard 11*s*. 6*d*. Otherwise: if any person will add to the given sum 3 shillings in favour of Henry to make him equal with Richard, the 23 shillings divided equally will give each

11*s.* 6*d.*; but if the 3 shillings had not been added, Henry would receive only 8*s.* 6*d.*

2. Questions in Arithmetic are often proposed by giving the sum and the difference of two unequal quantities, and requiring to find each quantity.

Ex. 1. At an election of one of two candidates 457 persons voted, and the successful candidate had a majority of 43 votes. How many voted for each candidate?

$\frac{1}{2}(457 + 43) = 250$ for the successful one;
$250 - 43 = 207$ for the other candidate. } *Ans.*

Ex. 2. A boatman can row $6\frac{1}{4}$ miles an hour down a river, but only 5 miles an hour up. What is the velocity of the current, and what is the man's power of rowing in still water?

The stream's rate added to the man's rate of rowing in still water $= 6\frac{1}{4}$ miles; but the man's rate diminished by the opposing rate of the stream $= 5$.

Hence $\frac{1}{2}(6\frac{1}{4} - 5) = \frac{5}{8}$ mile per hour for the stream;
$\frac{5}{8} + 5 = 5\frac{5}{8}$ miles per hour for the man. } *Ans.*

Ex. 3. The sum of the circumferences of two circles is 125·664, and the difference of their diameters is 12. Find the diameter of each, reckoning the circumference of a circle = 3·1416 of the diameter.

The sum of the diameters $= 125{\cdot}664 \div 3{\cdot}1416 = 40$; hence the diameters are

$\frac{1}{2}(40 \pm 12) = 26$ and 14. *Ans.*

Ex. 4. The less of two lengths is $7\frac{1}{2}$ yards, and their difference is equal to $\frac{3}{8}$ of their sum. Find the greater.

The sum of the quantities is 8 units of measure, and their difference 3 such units; therefore the greater quantity is to the less as $8+3 : 8-3$, or the greater quantity is equal to $\frac{11}{5}$ of the less; hence $\frac{11}{5}$ of $7\frac{1}{2}$ yards $= 33$ yards $\div 2 = 16\frac{1}{2}$ yards. *Ans.*

EXERCISES I.

1. Divide £13 10*s*. $7\frac{1}{2}d$. into two parts, one of which shall exceed the other by 13*s*. $10\frac{1}{2}d$.

2. A profit of £72 7*s*. 6*d*. is to be divided between two partners, A and B; what will A's share be, if he gets £23 2*s*. 6*d*. less than B?

3. The joint extent of two contiguous farms is 583 acres 1 rood 37 perches, and one of them is 67 acres 2 roods 33 perches less than the other. Find the acreage of each.

4. The sum of two quantities is $234\frac{5}{6}$, and their difference is $67\frac{8}{9}$. Find each of them.

5. Divide the fraction $\frac{1}{8}$ into two parts, one of them to be less than the other by $\frac{3}{5}\frac{4}{6}$.

6. Two trains, M and N, are running in opposite directions on parallel lines of a railway, and pass each other with the velocity of 74 miles an hour; were they running in one direction, M would pass N with the velocity of five miles an hour. What is the single rate of each?

7. The sum of two quantities is $5\frac{1}{6}$, and their difference is equal to the square root of $\cdot 18\dot{7}$. Find them.

8. A rectangular field, 348 yards longer than broad, is inclosed by 2600 yards of fencing. What are its dimensions?

9. Find two fractions of which the sum is equal to $\frac{7}{9} - \frac{5}{12}$, and the difference equal to $\frac{1}{10} + \frac{2}{15}$.

10. The perimeter of a triangle, A B C, is 235; the sides AB, BC, are together 123 greater than AC, and BC exceeds AB by 7. Find each side.

11. A and B speculate with equal sums; A's returns amount to £456 10*s*., and B's to £403 10*s*., A having gained exactly as much as B lost. What amount did each invest?

12. Two circles touch each other, the circumference of one being 333 longer than that of the other, and the distance between their centres is 134. Find the diameter of each, reckoning the circumference of a circle equal to 3·1416 times the diameter.

CHAPTER II.

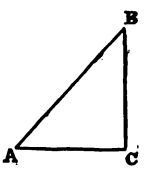
OF RIGHT-ANGLED TRIANGLES.

1. It is generally allowed that candidates in Higher Arithmetic ought to be familiar with, and at liberty to apply, the doctrine of Euclid I. 47 respecting right-angled triangles. And we would, moreover, advocate an allowance for such candidates to make free application of the doctrine that the product of the sum of two quantities by their difference is equal to the difference of the squares of the quantities—a doctrine made evident by the algebraic identity

$$(a + b)(a - b) = a^2 - b^2,$$

and corresponding to what is demonstrated in Euclid II. 5.

The latter doctrine may be verified by the right-angled triangle A B C, if the hypotenuse A B be given, and the base A C, to find C B. Let A B = 29, and A C = 20; then $BC^2 = AB^2 - AC^2 = 29^2 - 20^2 = 841 - 400 = 441$; and this result is also that of (A B + A C)(A B − A C), for (29 + 20)(29 − 20) = 49 × 9 = 441; whence B C = $\sqrt{441} = 21$.



2. If A C in the above triangle had been given = 19 instead of 20, the value of B C would have been equal to $\sqrt{(29 + 19)(29 - 19)} = \sqrt{480} = 21{\cdot}9089$, &c., the decimal part of this value being interminable.

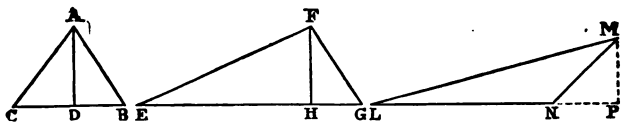
Now it is well to know how to form a right-angled triangle having sides of exact length. Either of two methods may be used.

First Method.—For the sides containing the right angle, take any two values, one of which is equal to $\frac{3}{4}$ of the other. In verification we have $(3x)^2 + (4x)^2 = (5x)^2$. This is familiar to many students.

Second Method.—For one of the sides containing the right angle take twice the product of any two unequal numbers; and for the other side take the difference of the squares of the same numbers. The hypotenuse of such a triangle will be equal to the sum of the squares of the two chosen numbers; for we have $(x^2 + y^2)^2 - (x^2 - y^2)^2 = 2x^2 \times 2y^2 = (2xy)^2$.

Thus, twice 7×3 and $7^2 - 3^2$ give us 42 and 40 for sides to contain the right angle, and the hypotenuse will be equal to $7^2 + 3^2 = 58$. In verification we have $42^2 + 40^2 = 3364 = 58^2$.

3. The above methods enable us to construct scalene triangles having exact areas.[1]



In the first of the above figures, take BD = twice $5 \times 2 = 20$, AD $= 5^2 - 2^2 = 21$, then AB will be $= 5^2 + 2^2 = 29$. Now produce BD to C, making DC $= \frac{4}{3}$ of AD $= 28$; then AC will be $= \frac{5}{3}$ of AD $= 35$; and thus we have a scalene triangle, CAB, with sides, AC $= 35$, AB $= 29$, CB $= 48$, and having an exact area $= \frac{1}{2}(48 \times 21) = 504$.

[1] The student should copy each of the figures following, and write the values of the lines upon it.

In the second figure, take E H = twice $9 \times 6 = 108$, F H $= 9^2 - 6^2 = 45$; then E F will be $= 9^2 + 6^2 = 117$. Now, we can here repeat the second method, for F H happens to be $= 7^2 - 2^2$; therefore, if H G be made = twice $7 \times 2 = 28$, we shall have F G $= 7^2 + 2^2 = 53$; and thus we have a scalene triangle E F G, with sides E F = 117, F G = 53, E G = 136, and having an exact area $= \frac{1}{2}(136 \times 45) = 3060$.

In the third figure, take L P = twice $11 \times 14 = 308$, P M $= 14^2 - 11^2 = 75$; then L M will be $= 14^2 + 11^2 = 317$. Then take P N $= \frac{4}{3}$ of $75 = 100$, and M N $= \frac{5}{3}$ of $75 = 125$; and thus we have a scalene triangle L M N, with sides L M = 317, M N = 125, L N $= 308 - 100 = 208$, and having an exact area $= \frac{1}{2}(208 \times 75) = 7800$.

4. When the base and the two other sides of a triangle are given, to find the segments into which the base is divided by the perpendicular upon it from the vertex: the difference of the squares of the two sides, divided by the base, gives the difference of the segments; and thus we have *sum* and *difference*, from which to find each segment.

Refer to the first of the preceding figures, where C A is 35, A B 29, and C B 48;

A D^2 = A C^2 − C D^2 = A B^2 − D B^2;

$\therefore$ A C^2 − A B^2 = C D^2 − D B^2 = (C D + D B)(C D − D B), that is $(35 + 29)(35 - 29) = 48$ (C D − D B);

$\therefore$ C D − D B $= \frac{64 - 6}{48} = 8$; but C D + D B = 48,

$\therefore$ C D $= \frac{1}{2}(48 + 8) = 28$, and D B $= 28 - 8 = 20$.

Another way of determining the segments is by first finding the perpendicular A D. The area of the triangle, according to a well-known rule, is

$= \sqrt{(56 \times 21 \times 27 \times 8)} = \sqrt{(56 \times 7 \times 81 \times 8)} = 56 \times 9$;

but we have also twice the area $= CB \times AD = 48\ AD$;

hence $AD = (56 \times 18) \div 48 = 21$;

and now, $CD = \sqrt{(CA^2 - AD^2)} = \sqrt{(56 \times 14)}$
$= \sqrt{(14^2 \times 4)} = 14 \times 2 = 28$.

Note.—If the triangle, as is generally the case, did not happen to have an exact area, the calculation would have been tedious.

Refer now to the third of the preceding figures, where the perpendicular falls without the base. Here $LN + NP$ and $-PN$ are to be regarded as the segments of LN. And for the calculation we have

$$(317^2 - 125^2) \div 208 = \frac{442 \times 192}{208} = 34 \times 12 = 408;$$

hence the difference of the segments $= LN + NP + PN$,
and the sum „ „ $= LN = 208$;

$$\therefore \tfrac{1}{2}(408 + 208) = 308 = LP.$$

5. When, in a right-angled triangle, one of the sides containing the right angle, or the ratio it bears to the other, and either the sum or the difference of the hypotenuse and that other side are known, the triangle is completely determined.

Ex. 1. The sum of the hypotenuse and base is 338, and the other side is 208. Find the hypotenuse.

Refer to the figure in § 1:—We have $BC = 208$, and $AB + AC = 338$;
now, $AB^2 - AC^2 = BC^2$, or $(AB + AC)(AB - AC) = BC^2$, or $338(AB - AC) = 208^2$;
hence $AB - AC = (208 \times 208) \div 338 = \frac{1}{2}(16 \times 16) = 128$;
but $AB + AC = 338$; hence $\frac{1}{2}(338 + 128) = 233$. *Ans.*

Ex. 2. The breadth of a rectangular floor is equal

to half the length, and is $12\frac{3}{4}$ feet shorter than the diagonal. What is the length of the floor?

Take unity to represent the breadth, then 2 represents the length, and $\sqrt{5} = 2{\cdot}236068$ the diagonal, which exceeds unity by $1{\cdot}236068$;
hence $(12{\cdot}75 \text{ ft.} \div 1{\cdot}236068) \times 2 = 20{\cdot}63$ ft. *Ans.*

6. When two given sides of a right-angled triangle contain a common factor, it may be left out, and a proportionate third side found, which is then to be multiplied by the reserved factor.

Example.—The hypotenuse of a right-angled triangle is 455, and one of the other sides is 231. Find the remaining side.

Here by cancelling 7 we find the given sides proportioned as 65 : 33;
$\sqrt{(98 \times 32)} = \sqrt{49} \times \sqrt{64} = 7 \times 8 = 56$;
hence $56 \times 7 = 392$. *Ans.*

7. When the acute angles of a right-angled triangle are equal, that is, each $= 45°$, the two sides containing the right angle are equal, and each of them is to the hypotenuse as $1 : \sqrt{2}$, or nearly as $1 : 1{\cdot}4142$.

When one of the acute angles is 60°, the other is 30°, the shorter of the two sides containing the right angle is equal to half the hypotenuse, and therefore the hypotenuse is to the longer of the two as $2 : \sqrt{(4-1)}$, that is, as $2 : \sqrt{3}$, or as $1 : \frac{1}{2}\sqrt{3}$, or *nearly* as $1 : {\cdot}866$.

Ex. 1. The hypotenuse of an isosceles right-angled triangle is 239. Find each of the equal sides.

$$\frac{239}{\sqrt{2}} = \frac{239 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \tfrac{1}{2}\sqrt{2} \times 239 = {\cdot}7071 \times 239$$
$= 169$ *nearly*. *Ans.* (See Ch. V. Ex. 10.)

Ex. 2. The side of an equilateral triangle is 515; required its perpendicular altitude.

$\frac{1}{2}\sqrt{3} \times 515 = \cdot 866 \times 515 = 446$ *nearly.* *Ans.*

EXERCISES 2.

1. Moscow is about 400 miles south-east from Petersburg, and about 500 north-east from Kiev. How far is Kiev from Petersburg?

2. The perimeter of a rectangle is 1106, and the length exceeds the breadth by 263. Find its diagonal.

3. Find the area of a right-angled triangle, one of the sides containing the right angle being 15 feet, and the other 50 inches less than the hypotenuse?

4. A pole A B, 21 feet high, standing on level ground, is *broken over* by the wind, and the top, B, in falling, strikes the ground at a distance of 9 feet from A. What is the height of the part left standing?

5. L expects a visit from M and N, the former of whom lives a short distance south of N, and 272 yards east of L. If M, on leaving home, goes direct to N's house, and then with N direct to L's house, he will have walked 578 yards. How far is L's house from N's?

6. What is the length of each side of a square, when 10 inches shorter than the diagonal?

7. The top of a ladder just reaches the top of a wall, when its foot is at a horizontal distance of 10 feet from the bottom of the wall; but if the foot of the ladder be drawn 4 feet farther from the wall, its top will reach a point 2 feet below the top of the wall. What is the length of the ladder?

8. In a right-angled triangle, one of the sides con-

taining the right angle is equal to three-fifths of the other, and the hypotenuse is 35 inches. Find the area of the triangle.

9. In a right-angled triangle, one of the sides containing the right angle is 60, and the other is equal to $\frac{5}{9}$ of the hypotenuse. Find the hypotenuse.

10. The sides of a triangle are 221, 368, and 285; what is its area and the distance of the longest side from the opposite vertex?

11. The sides of a triangle are 281, 385, and 468. Find the segments into which the longest side is divided by a perpendicular on it from the opposite vertex.

12. The sides of a triangle are A B 2788, B C 925, A C 2133. Find the distance of A C from B.

13. The base A C of the triangle A B C is 171, and the other sides are A B 109, and B C 100. Find the altitude B D, and the segments A D, D C.

14. In a right-angled triangle, the greater of the perpendicular sides is 35, and their difference is equal to $\frac{4}{9}$ of their sum. Find the hypotenuse.

15. The area of an oblique parallelogram A B C D is 956, the side A B being 26, and B C D an angle of 45°. Find the length A D.

16. Find the area of an oblique parallelogram A B C D, the side A B being 14 inches, the length B C 23 inches, and the angle A B C 120°.

CHAPTER III.

QUESTIONS ON COMBINATIONS OF QUANTITIES, ACCORDING TO THE RATIO OF THEIR SINGLE VALUES.

THE most elementary form of the subject now proposed may be exemplified by the following problem:—

Ex. 1. A can perform a piece of work in 27 hours, B can do it in 30 hours; in what time could A and B together do it?

Suppose the whole work to consist of some common multiple of 27 and 30, say 270 units of work.

In one hour A does 10 units and B 9 units; 19 units together; hence the two could do the whole work in $270 \div 19 = 14\frac{4}{19}$ hours. *Ans.*

Now, the same problem may be stated in the following different form:—

Ex. 2. If 3 men or 5 women could reap a field in 18 hours, in what time could 2 men and 3 women reap it?

Here, as 3 men would take 18 hours, 1 man would take 54 hours, and 2 men would take 27 hours.

Also, as 5 women would take 18 hours, 1 woman would take 90 hours, and 3 women would take 30 hours.

Now, if the whole work be supposed to consist of 270 units, then, in one hour, 2 men would do 10 units and 3 women 9 units; so that 2 men and 3 women

together would do 19 units per hour, or the whole 270 units in $14\frac{4}{19}$ hours. *Ans.*

Notice also the virtual sameness of the next two problems.

Ex. 3. A and B together can do a piece of work in $6\frac{1}{4}$ days, which B by himself would do in $13\frac{1}{3}$ days; in what time could A alone perform it?

Suppose the whole work to consist of some common multiple of $6\frac{1}{4}$ and $13\frac{1}{3}$, say 200 units.

In one day A and B together do 32 units, and B alone 15 units; hence A alone 32−15 or 17 units; so that A's time for the whole 200 units would be $11\frac{13}{17}$ days. *Ans.*

Ex. 4. If 4 men and 6 women together can do in $6\frac{1}{4}$ days a piece of work which 15 women could do in $5\frac{1}{3}$ days, in what time could the 4 men alone do it?

Here, as 15 women would take $5\frac{1}{3}$ days, 1 woman would take 80 days, and 6 women $13\frac{1}{3}$ days.

Now, if the whole work be supposed to consist of 200 units, then, in 1 day 4 men and 6 women together do 32 units, and 6 women do 15 units; hence the 4 men do 32−15 or 17 units, and would do the whole 200 units in $11\frac{13}{17}$ days. *Ans.*

*** Problems, however, stated in the manner of Examples 1 and 3 deserve to be considered apart from the others contained in the present chapter. They form the subject of Chapter IV.

Ex. 5. If, to execute a certain piece of work, 8 men and 7 youths would take 33 hours, but 6 men and 9 youths would require 35 hours, what time would 12 men and 7 youths take?

The work would be done in 1 hour,
either by (8m.+7y.)×33, or by (6m.+9y.)×35;

so that we have 264m. + 231y. = 210m. + 315y.,
which implies 54 men = 84 youths, or $4\frac{1}{2}$ men = 7 youths;
hence 12 men with 7 youths = $16\frac{1}{2}$ men, and 8 men with 7 youths = $12\frac{1}{2}$ men.
The question now is: If $12\frac{1}{2}$ men would take 33 hours, what time would $16\frac{1}{2}$ men take?
1 man would take 33 hrs. $\times 12\frac{1}{2}$,

$$\therefore 16\tfrac{1}{2} \text{ men would take } \frac{33 \text{ hrs.} \times 12\frac{1}{2}}{16\frac{1}{2}} = 25 \text{ hrs. } \textit{Ans.}$$

Ex. 6. I invested part of £500 in a speculation by which I gained 10 per cent.; but by investing the remainder in another speculation I lost $6\frac{1}{4}$ per cent., thereby reducing my gain on the whole to $4\frac{4}{5}$ per cent. Find the particular investments.

$1st \times 10 - 2nd \times 6\frac{1}{4} = 1st \times 4\frac{4}{5} + 2nd \times 4\frac{4}{5}$;
$\therefore (10 - 4\frac{4}{5})$ of the $1st = (4\frac{4}{5} + 6\frac{1}{4})$ of the $2nd$;
or $1st \times 5\frac{1}{5} = 2nd \times 11\frac{1}{20}$, or $1st \times 26 \times 4 = 2nd \times 221$;
or 8 times the $1st$ = 17 times the $2nd$,
or the $2nd = \frac{8}{17}$ of the $1st$; and hence $1st$ and $2nd$ together = $1\frac{8}{17}$ of the $1st$.
$\therefore$ £500 $\div 1\frac{8}{17}$ = £20 × 17 = £340, the $1st$ investment, leaving £160 the $2nd$. *Ans.*

Ex. 7. A house cost 3 times as much for materials as for labour. Had the materials cost $7\frac{1}{2}$ per cent. more, and the labour 5 per cent. less, the cost of the house would have been £2087 10*s*. What was its cost?

Call the cost of labour L; that of materials 3L.

$$\frac{107\frac{1}{2}}{100} \text{ of } 3L + \frac{95}{100} \text{ of } L = £2087{\cdot}5$$

$$(322{\cdot}5 + 95) \text{ of } L = £208750$$

$$\therefore L = £2087500 \div 4175 = £500;$$

$$\therefore 4L = £2000. \quad \textit{Ans.}$$

Ex. 8. An ornament is made of gold and silver; the

weight of the gold used is equal to $\frac{2}{25}$ of that of the silver, and the cost is £14 13*s*. Assuming the volume of gold to be equal to $\frac{1}{3}\frac{2}{2}$ of that of an equal weight of silver, and the value of gold to be $17\frac{1}{2}$ times the value of an equal volume of silver, find the values of the gold and silver, respectively, used in making the ornament.

The gold used weighs 2 units, and the silver 25 such units. Let the unit of weight be called W.

25W of silver are equal in volume to $\frac{25 \times 32}{19}$W of gold;

and equal in value to $\frac{25 \times 32}{19 \times 17\frac{1}{2}}$W of gold;

but the value of 2W of gold + 25W of silver = 293*s*.,

that is, $2\text{W} + \frac{25 \times 32}{19 \times 17\frac{1}{2}}$W of gold cost 293*s*.,

or $\frac{586}{133}$W of gold cost 293*s*., ∴ 2W of gold cost 133*s*.;

hence the value of the gold used is £6 13*s*., and that of the silver £8. *Ans.*

Ex. 9. A farm is let for £96 and the value of a certain number of quarters of wheat. When wheat is 38*s*. a quarter, the whole rent is 15 per cent. lower than when it is 56*s*. a quarter. Find the number of quarters of wheat that forms part of the rent.

Call the number of quarters of wheat W.

$1920 + 38\text{W} = \frac{85}{100}$ of $(1920 + 56\text{W})$;

or $(960 + 19\text{W}) \times 20 = (960 + 28\text{W}) \times 17$,

or $(960 + 19\text{W}) \times 5 = (240 + 7\text{W}) \times 17$;

hence $119\text{W} - 95\text{W} = 4800 - 4080$;

$24\text{W} = 720$; ∴ W = 30 quarters. *Ans.*

*** It should be noticed that the method used in the preceding questions will serve for the solution of

certain Alligation Problems frequently proposed. The next two examples are of this kind.

Ex. 10. In what proportions must spirits worth 16*s.* 8*d.* and 17*s.* 6*d.* a gallon be mingled, to make a blend worth 17*s.* a gallon?

$17s. \times 1st$ no. $+ 17s. \times 2nd$ no. $= 16\frac{2}{3}s. \times 1st + 17\frac{1}{2}s. \times 2nd$;
$\therefore (17 - 16\frac{2}{3}) \times 1st = (17\frac{1}{2} - 17) \times 2nd$,
or the $1st$: the $2nd$:: $\frac{1}{2} : \frac{1}{3}$ or as 3 : 2. *Ans.*

Ex. 11. Divide 53*s.* 6*d.* into two such parts, that 14 times one of them added to 17 times the other will amount to £42 15*s.*

$1st \times 14 + 2nd \times 17 = 855s.$
also, $1st \times 17 + 2nd \times 17 = 909\frac{1}{2}s.$
hence, $1st \times 3 = 54\frac{1}{2}s.$, or the $1st$ part is 18*s.* 2*d.*
and the $2nd$ „ 35*s.* 4*d.* } *Ans.*

EXERCISES 3.

1. When 16 lbs. of tea and $12\frac{3}{4}$ lbs. of coffee are together equal in value to $2\frac{7}{12}$ dozen of wine, and 5 lbs. of tea are worth as much as 12 lbs. of coffee, how many lbs. of coffee are equal in value to 15 dozen of wine?

2. If 16 men or 20 women can reap a field in a day of 9 hours, in what time could 1 man and 1 woman do the same?

3. If 7 men and 23 boys can do a piece of work in 9 hours, in what time can 23 men and 7 boys do it, if the ability of a man be thrice that of a boy?

4. If 10 women, or 12 boys, or 15 girls, can do a piece of work in 1 day, in what time could 1 woman, 1 boy, and 1 girl together do it?

5. A money changer gives £4 7*s.* 1*d.* for 13 coins

of one sort along with 8 of another, and he gives the same value for 9 of the former along with 14 of the latter. What value does he give for each coin?

6. The sum of £34 16*s.* is to be divided among 14 men and 13 women, each man's share to be in such proportion to a woman's, that a man and woman may together receive £2 11*s.* Find the share of each man.

7. When 2 sofas, 3 tables, and 9 chairs together cost £63 16*s.*, what will a sofa, together with a table and 4 chairs come to, if a sofa be worth 2 tables, and a table worth 5 chairs?

8. A field can be reaped by 2 men and 6 women in 7 days, or by 5 men and 8 women in 4 days. In what time could 6 men and 4 women reap the field?

9. In what proportions must tobaccos at 4*s.* 9*d.* and 5*s.* 8*d.* a lb. be mixed that the mixture may cost 5*s.* a lb.?

10. Divide the fraction $\frac{17}{18}$ into two such parts that 8 times one of them added to 11 times the other may make $7\frac{8}{9}$.

11. Divide the number 150 into two parts, so that 16 times the greater may exceed 19 times the less by 55.

12. What two decimal aggregate parts of unity are such that $\frac{3}{4}$ of the greater added to $\frac{5}{6}$ of the less make ·77?

13. If 3 men can earn as much in a day as 5 women, and 6 women as much as 11 boys; and if 5 men, 7 women, and 6 boys, earn together £20 9*s.* 4*d.* in 8 days; how much will 7 men, 6 women, and 10 boys earn together in 10 days?

14. If 6 men and 5 women can cut down a crop in $9\frac{9}{11}$ days, whereas 5 men and 6 women would require

$10\frac{2}{7}$ days; what are the smallest exact numbers of men and women corresponding in ability?

15. What was the price of cod per lb., and of oysters per dozen, when the proprietor of a restaurant paid 69*s.* for 58 lbs. of cod and 17 dozen oysters, whereas now, when cod is $8\frac{1}{3}$ per cent. cheaper and oysters $6\frac{1}{4}$ per cent. dearer, he is charged 50*s.* 3*d.* for 36 lbs. of cod and 16 dozen oysters?

16. What are the respective prices of beef and mutton per stone, if a rise of $6\frac{2}{3}$ per cent. in the price of beef along with a fall of 5 per cent. in mutton would make 12 stone of beef with 10 of mutton cost £7 3*s.* 4*d.*, whereas a fall of 4 per cent. in beef along with a rise of $6\frac{1}{4}$ per cent. in mutton would make 5 stone of beef with 6 of mutton cost £3 12*s.* 6*d.*?

CHAPTER IV.

QUESTIONS RELATING TO THE JOINT WORK OF AGENTS OF DIFFERENT POWERS.

In the preceding chapter we introduced two examples of the kind which we now propose farther to illustrate, according to a general mode of treatment that is not conveniently applicable to most of the other questions in that chapter.

Ex. 1. A can do a piece of work in 18 days, B the same in 21 days, and C the same in 28 days. What time would A, B, and C together take to do it?

Suppose the whole of the work to consist of some common multiple of 18, 21, and 28, say 252 units:

A does 14 units per day, B 12, C 9; 35 units together;

∴ the three together would do the whole in $252 \div 35 = 7\frac{1}{5}$ days. *Ans.*

Ex. 2. A water cistern has a supply pipe that can fill it in 33 minutes, and a discharge pipe that can empty it in 57 minutes. If, when the cistern is empty, the two pipes are set open together, in what time will it be full?

Suppose the contents of the cistern to be some common multiple of 33 and 57, say 627 units:

In 1 minute, 19 units flow in, and 11 run out;

so that 8 units are gained by the supply pipe per minute;

Hence the time required $= 627 \div 8 = 78\frac{3}{8}$ min. *Ans.*

Ex. 3. A, with B's assistance, could complete a piece of work in 9 hours, or, with C's assistance the time of completion would be 12 hours; but with the assistance of both B and C, the time would be only 8 hours. In what time could each person singly complete the piece of work?

Suppose the whole of the work to consist of some common multiple of 9, 12, and 8, say 72 units:

A, B, and C together can do 9 units per hour, A with B 8 units, A with C 6 units;

$\therefore$ B can do $9-6$, or 3 units, per hour, and C can do $9-8$, or 1 unit, per hour; and hence A can do $9-3-1$, or 5 units per hour. Accordingly,

A's time for the whole $= 72 \div 5 = 14\frac{2}{5}$ hours
B's " " $= 72 \div 3 = 24$ "
C's " " $= 72 \div 1 = 72$ "
} *Ans.*

Ex. 4. A begins by himself a piece of work which he can complete in 15 days; but after three days he is joined by B, and in $6\frac{2}{3}$ days more the work is completed. In what time could B alone have done the whole?

Suppose the whole work to consist of a common multiple of 15 and $6\frac{2}{3}$, say 60 units:

A does 4 units per day, and in 3 days does 12 units, leaving 48 units, which are done by A and B together in $6\frac{2}{3}$ days;

hence $48 \div 6\frac{2}{3} = 7\frac{1}{5}$ units per day by A and B jointly.

But A does 4 units per day, $\therefore$ B $3\frac{1}{5}$, and hence

$60 \div 3\frac{1}{5} = 18\frac{3}{4}$ days. *Ans.*

Ex. 5. A certain piece of work is done by L, M, and N. The whole could be done by L alone in 14 hours, by M alone in $12\frac{1}{4}$ hours, or by N alone in $15\frac{3}{4}$ hours. L and M begin it together 3 hours before N joins them, but M goes away 1 hour 20 minutes before the actual completion of the work. In what time is the work done?

Suppose the whole work to consist of some common multiple of 14, $12\frac{1}{4}$, and $15\frac{3}{4}$, say 882 units;

L does 63 units per hour, M 72, N 56.

Now, if M and N had not been away, they would have added $72 \times 1\frac{1}{3} + 56 \times 3 = 264$ units to the work, extending it to $882 + 264$, or 1146 units, done by L, M, and N together, at the rate of $63 + 72 + 56 = 191$ units per hour;

hence $1146 \div 191 = 6$ hours. *Ans.*

Ex. 6. A, B, C, and D together do a piece of work for which A by himself would require 2 hours less than B. Now A and B together could do it in $\frac{50}{81}$ of the time C and D together would take, A and C in $\frac{56}{75}$ of the time B and D would take, and B and C in $\frac{65}{66}$ of the time A and D would take. In what time would each person singly do the piece of work?

The time in which the whole work is actually done is that of A, B, C, and D, working together. Now if, to do the whole, A and B would take $\frac{50}{81}$ of the time of C and D,

then, A and B would take $1\frac{50}{81}$ of the time of A, B, C, and D;

similarly, A and C would take $1\frac{56}{75}$ of the time of A, B, C, and D;

and B and C would take $1\frac{65}{66}$ of the time of A, B, C, and D.

Hence A and B do $\frac{81}{131}$ of the whole, A and C $\frac{75}{131}$, B and C $\frac{66}{131}$; $\therefore$ A, B, and C do $\frac{1}{2}\left(\frac{81+75+66}{131}\right)$, or $\frac{111}{131}$; so that the portions of the work done by each are as 111 − 66, 111 − 75, 111 − 81, and 131 − 111, or as 45, 36, 30, and 20; and therefore the times in which they could singly do the whole are as $\frac{1}{45}$, $\frac{1}{36}$, $\frac{1}{30}$, and $\frac{1}{20}$, or as 4, 5, 6, 9; but B takes two hours longer than A, which is therefore $\frac{1}{4}$ longer than A;

hence A's time = 2 hours × 4 = 8 hours
B's " = $\frac{5}{4}$ of A's = 10 "
C's " = $\frac{6}{5}$ of B's = 12 "
D's " = $\frac{3}{2}$ of C's = 18 "

Ans.

EXERCISES 4.

1. A can do in 6 days a piece of work which would occupy B for 8 days. In what time could A and B together do it?

2. A and B together perform a piece of work in $5\frac{5}{6}$ hours, but A alone would take 10 hours to perform it. What time would B take?

3. To complete an excavation, A by himself would take 18 days, B 21 days, C 28 days. What time would A, B, and C together take?

4. A water cistern has a supply pipe, A, that fills it in 63 minutes, and it has two discharge pipes, B and C, the first of which can empty the cistern in 72 minutes, and the second in 80 minutes. If, when the cistern is full, all three pipes are set open together, in what time will it be empty?

5. Three taps can fill a cistern, A by itself in 24 minutes, B in 20 minutes, C in 27. They are all three turned on at once, when the cistern is empty, and after

running for $4\frac{1}{2}$ minutes, B and C are turned off. How long will A then take by itself to complete the filling of the cistern?

6. A cistern can be filled by its supply pipe in 1 hour 12 minutes; but a leak at the bottom causes the actual time of filling to be 1 hour 36 minutes. If the supply be stopped when the cistern is $\frac{3}{4}$ full, in what time will it be emptied?

7. A, B, C, and D undertake to do a piece of work. A's rate of working is to B's as 3 to 4, B's is to C's as 5 to 4, and C's is to D's as 5 to 6. A would do it alone in 60 hours. Find in what time each of the other three would do it alone, and in what time all four would do it working together.

8. The area of a side of a cubical cistern is $12\frac{1}{4}$ feet; it is furnished with three taps, one of which could fill the cistern in 10 minutes, another could fill it in 12 minutes, and the third could empty it in 15 minutes. If all three be opened, the cistern being empty, find the weight of the water in the cistern at the end of 5 minutes: a cubic foot of water = 1000 oz.

9. A could do a piece of work in $\frac{5}{6}$ of the time that B would take; and the two working together would do it in $3\frac{3}{4}$ days. How long would each by himself take to do it?

10. A piece of work is done by A, B, and C, as follows:—A begins it, and is joined by B at the middle of the first day, and by C at the middle of the second day. In what time is the work done, if A alone could have done it in 14 days, B in 21, C in 20?

11. A alone can do in 7 days as great an amount of work as B and C together can do in 5 days, and B alone can do in 11 days as much as A and C together can do

in 5 days. Compare the rates of working of A, B, and C.

12. A alone could have done a certain piece of work in 25 days, B the same in 27 days, C the same in 30 days. The work, however, was done by the three, all of them beginning it together; but B absented himself 6 days, and C 13 days, during the progress of the work. In what time was it completed?

13. A contractor employs 16 equally qualified men to do a certain work; but after they have wrought for 6 days, 3 of them refuse to work any longer, and 8 days later, 2 men are obliged by illness to leave the work. It is completed, however, in 24 days from the commencement. How long would it have taken if all had wrought the full time?

14. A piece of work is performed by A and B, of whom the former could do the whole by himself in 10 hours, the latter by himself in 14 hours. They begin the work together, but B leaves off 2 hours before the completion of the work. In what time is the work done?

15. A certain piece of work was executed by A, B, and C, of whom A by himself could have done the whole in 16 days, B the whole in 18 days, C the whole in 22 days. In what time was the work completed, C having gone away $4\frac{1}{3}$ days, and B 2 days before the completion?

16. A piece of work is to be executed by A, B, and C, working together. In what time could each man singly perform the whole, if A and B together would do it in $\frac{1}{3}$ of the time that C would take, and B and C in $\frac{5}{7}$ of the time that A would take, and supposing that B alone would take $2\frac{1}{4}$ hours longer than A alone to do the whole?

CHAPTER V.

PROPORTIONATE AND IRREGULAR DISTRIBUTIONS OF QUANTITIES.

(*i.*) *Proportionate Distribution.*

1. The distribution of a quantity into two or more proportional parts is a process frequently required in miscellaneous problems, and expertness in the management of it is a very important qualification. The most elementary form of its application may be exemplified by the distribution of the number 999 into three parts in the proportion of 5 : 8 : 14.

Here the parts required are to be such that a fifth of the first, an eighth of the second, and a fourteenth of the third shall be identical quantities; or, as the sum of the given distributives is 27, and the first is $\frac{5}{27}$ of that sum, the second $\frac{8}{27}$ of it, and the third $\frac{14}{27}$, so are to be the relations of the required quantities; and, accordingly, to solve the problem, we have

$\frac{5}{27}$ of $999 = 37 \times 5 = 185$, the 1st part,
$\frac{8}{27}$ of „ $= 37 \times 8 = 296$, the 2nd „
$\frac{14}{27}$ of „ $= 37 \times 14 = 518$, the 3rd „

2. The student should notice also other forms of relation involved in proportionate distribution, such, for instance, as the following:—

(i.) The 1st part will be equal to $\frac{5}{8}$ of the 2nd

Verification, $185 = \frac{5}{8}$ of $296 = 37 \times 5$.

(ii.) The sum of the 1st and 2nd parts will be equal to $\frac{5+8}{27}$ of the whole.

Verification, $185 + 296 = 481 = \frac{13}{27}$ of $999 = 37 \times 13$

(iii.) The difference of the 3rd and 1st parts will be equal to $\frac{14-5}{8}$ of the 2nd.

Verification, $518 - 185 = 333 = \frac{9}{8}$ of $296 = 37 \times 9$.

(iv.) The difference of the 3rd and 2nd parts will be equal to $\frac{14-8}{5+8}$ of the sum of the 1st and 2nd parts.

Verification, $518 - 296 = 222 = \frac{6}{13}$ of $(185 + 296)$ $= \frac{6}{13}$ of $481 = 37 \times 6$.

3. The distributives in the preceding question express a direct proportion, the proposition being to find three numbers amounting together to 999, and such that the first *divided* by 5, the second by 8, and the third by 14, shall give equal results.

But distributives are sometimes expressed by *multipliers* giving equal products. For example, it may be proposed to divide the number 999 into three such parts, that 5 times the first, 8 times the second, and 14 times the third shall be equal. The proportion now is not $5 : 8 : 14$, but $\frac{1}{5} : \frac{1}{8} : \frac{1}{14}$, because to divide by $\frac{1}{5}$ is the same as to multiply by 5, &c. Now, $\frac{1}{5}$, $\frac{1}{8}$, and $\frac{1}{14}$, reduced to a common denominator, have the numerators changed to 56, 35, and 20, so that the required parts will be as $56 : 35 : 20$.

Another way of obtaining this direct proportion may be used; for as the first of the required quantities multiplied by 5 is to be equal to the second multiplied

by 8, if we represent the first by 1, we must represent the second by $\frac{5}{8}$, in order that 5 times the first may be equal to 8 times the second. Again, if 1 is to represent the first, $\frac{5}{14}$ must represent the third, in order to satisfy the condition that 5 times the first and 14 times the third are to be equal. Accordingly, the direct proportion is $1 : \frac{5}{8} : \frac{5}{14}$, or $56 : 35 : 20$. And now the sum of these numbers being 111, or $\frac{1}{9}$ of 999, the remainder of the solution will be simply to multiply the distributives by 9, giving 504, 315, and 180, which are the parts required.

We will now present a few worked examples of the usual forms in which the more difficult questions in Proportionate Distribution are proposed.

Ex. 1. Find a vulgar fraction $= \frac{3}{17}$, and having the numerator 322 less than the denominator.

The numerator of the required fraction will, of course, be to its denominator as 3 : 17; hence the difference of the terms is equal to $\frac{17-3}{3}$ of the numerator, that is, $\frac{14}{3}$ of the numerator is 322, and $\frac{1}{3}$ of the numerator is 23; accordingly, the required fraction is $\frac{69}{391}$. *Ans.*

Ex. 2. Suppose a man leaves his estate to be divided among his three sons and two daughters in the following way:—The eldest son is to have one-third of the property, the second son a sum equal to two-thirds of the eldest's share, and the third a sum equal to four-fifths of the second's. The elder daughter's share is to be equal to two-fifths of what the second and third sons together receive, and the younger daughter is to have the remainder. It is found by this division that

the younger daughter will receive £520. Find the value of the estate.

Of the whole estate the eldest son is to have $\frac{1}{3}$, the second son $\frac{2}{3}$ of $\frac{1}{3} = \frac{2}{9}$, the third son $\frac{4}{5}$ of $\frac{2}{9} = \frac{8}{45}$, and the elder daughter $\frac{2}{5}$ of $(\frac{2}{9} + \frac{8}{45}) = \frac{4}{25}$.

The sum of these shares $= \frac{1}{3} + \frac{2}{9} + \frac{8}{45} + \frac{4}{25} = \frac{67}{75}$, leaving $\frac{8}{75}$ to the younger daughter = £520; therefore the whole estate £65 × 75 = £4875. *Ans.*

Ex. 3. Divide £52 6*s.* 8*d.* into three parts, the first of which shall be equal to 8 per cent. more than the second, and to 7 per cent. less than the third.

If 1 represent the 1*st* part, $\frac{100}{108}$, or $\frac{25}{27}$, represents the 2*nd*, and $\frac{100}{93}$ the 3*rd*.

These are as 837, 775, and 900; sum 2512;

£52$\frac{1}{3}$ ÷ £2512 = $\frac{1}{48}$ of each distributive,

= £17 8*s.* 9*d.*, £16 2*s.* 11*d.*, and £18 15. *Ans.*

Ex. 4. A table has a semicircular top, the perimeter of which is 20 feet. Find the length of the straight edge, the circumference of a circle being equal to 3·1416 multiplied by the diameter.

The length of the straight edge is to that of the curved edge as 1 : $\frac{1}{2}$ of 3·1416, or as 10000 : 15708; therefore the straight edge = $\frac{10000}{25708}$ of 20 feet = 7·78 feet. *Ans.*

Ex. 5. A's present age is to B's as 7 : 9; but 15 years ago A's age was to B's as 8 . 11. How old is A?

The difference of their ages now = $\frac{2}{7}$ of A's;

that of their ages 15 years ago = $\frac{3}{8}$ of A's;

but the difference is always the same quantity; therefore A's present age is to his former age as $\frac{3}{8} : \frac{2}{7}$, or as 21 : 16;

hence $\frac{5}{21}$ of A's = 15, and A's = 3 × 21 = 63. *Ans.*

Ex. 6. Divide £552 10*s.* among A, B, C, and D, so that B's share may be equal to $\frac{3}{14}$ of A's, C's equal to $\frac{7}{10}$ of B's, and the sum of A's and B's equal to $\frac{5}{8}$ of the sum of C's and D's.

If 14 represent A's share, 3 represents B's, and $2\frac{1}{10}$ represents C's.

Now, as we have 17 for the sum of A and B, and $2\frac{1}{10} \times \frac{5}{8}$ or $1\frac{5}{16}$ for $\frac{5}{8}$ of C, it appears that $\frac{5}{8}$ of D is represented by $17 - 1\frac{5}{16}$, or $15\frac{11}{16}$; and hence $15\frac{11}{16} \div \frac{5}{8} = 25\frac{1}{10}$ represents D.

The distributives, therefore, for A, B, C, D, respectively, are 14, 3, $2\frac{1}{10}$, $25\frac{1}{10}$, or 140, 30, 21, and 251; sum, 442; then £552½ ÷ 442 = £5 ÷ 4; hence £1¼ multiplied by the several distributives gives £175, £37 10*s.*, £26 5*s.*, and £313 15*s.* *Ans.*

Ex. 7. A certain sum is so divided among A, B, C, D, that A and D together receive £15. Moreover, A's share is equal to $\frac{4}{5}$ of B's, and C's is equal to $\frac{6}{13}$ of B's, and equal also to $\frac{5}{8}$ of D's. Find the share of each person.

Let A's share be represented by 1, then B's is $\frac{5}{4}$, C's is $\frac{15}{26}$, and D's is $\frac{8}{5}$ of $\frac{15}{26} = \frac{12}{13}$; hence $1\frac{12}{13}$ of A's = £15;

∴ A's = £7 16*s.*, B's = £9 15*s.*, C's = £4 10*s.*, D's = £7 4*s.* *Ans.*

Ex. 8. Divide £522 10*s.* into three parts, so that the simple interest, at 5 per cent., on the first for 48 days, that on the second for 54 days, and that on the third for 64 days may be all equal.

Here the meaning is that each principal multiplied by its number of days $\times \frac{5}{365\,00}$ will give the same result; but that fractional multiplier, being common to all the parts, may be cancelled, and we have simply to

distribute the given sum into such parts that 48 times the first, 54 times the second, and 64 times the third shall be equal. Accordingly, the parts may be represented by 1, $\frac{48}{54}$, and $\frac{48}{64}$, or by 1, $\frac{8}{9}$, and $\frac{3}{4}$, or by 36, 32, and 27; sum, 95. Then, £522$\frac{1}{2}$ ÷ 95 = £$5\frac{1}{2}$, which, multiplied by the several distributives, gives £198, £176, and £148 10*s*. *Ans*.

Ex. 9. Divide £1740 into three such parts, that the simple interest on the first at $3\frac{1}{2}$ per cent. per annum, for 3 years, may be equal to that on the second at $3\frac{3}{4}$ per cent. for 2 years, and equal also to that on the third at $5\frac{1}{4}$ per cent. for 4 years.

Here the meaning is, that the first part multiplied by $3\frac{1}{2} \times 3$ will be equal to the second multiplied by $3\frac{3}{4} \times 2$, and equal also to the third multiplied by $5\frac{1}{4} \times 4$; these multipliers are as 7, 5, and 14, and therefore the parts will be as $\frac{1}{7} : \frac{1}{5} : \frac{1}{14}$, or as 10 : 14 : 5; sum, 29; accordingly, £1740 ÷ 29 = £60, which being multiplied by the several distributives gives £600, £840, £300. *Ans*.

Ex. 10. What is the perpendicular altitude of an equilateral triangle, when it is 4 inches shorter than the side?

When the side of an equilateral triangle is 2, the perpendicular altitude is $\sqrt{3}$, and the altitude is $2-\sqrt{3}$ shorter than the side; what then will the altitude be when it is 4 inches shorter than the side? It will be equal to $\frac{4\sqrt{3}}{2-\sqrt{3}}$. Now this expression is a form not convenient for calculation; but, bearing in mind that the difference of any two abstract quantities multiplied by their sum gives a product equal to the difference of the squares of the quantities (see Chap. II. 7, Ex. 1), we should *rationalise*, as it is called, the

denominator of the fraction by multiplying both terms by $2+\sqrt{3}$, making the numerator $= 8\sqrt{3}+12$, and the denominator $= 2^2 - (\sqrt{3})^2$, or $4 - 3, = 1$; accordingly, $1{\cdot}732 \times 8 + 12 = 25{\cdot}856$ inches. *Ans.* (See Chap. II. 7, Ex. 1.)

Ex. 11. Divide £1560 10*s.* into three parts, so that their *amounts*, by simple interest, for $2\frac{1}{3}$ years at 5 per cent. per annum, $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent., and 4 years at 4 per cent., respectively, shall be equal.

The amounts of 100 are here $111\frac{2}{3}$, $108\frac{3}{4}$, 116, which are as 1340, 1305, and 1392; and we are to divide the given sum into parts which, multiplied by these three numbers respectively, shall give equal products. The parts will therefore be as $\frac{1}{1340}$, $\frac{1}{1305}$, and $\frac{1}{1392}$, or as 1044, 1072, 1005; sum, 3121; then £1560½ ÷ 3121 = £$\frac{1}{2}$, which multiplied by the several distributives gives £522, £536, £502 10*s.* *Ans.*

EXERCISES 5.

1. A certain sum is divided among four persons, A, B, C, D, in proportion as $\frac{4}{9} : \frac{5}{6} : \frac{3}{8} : \frac{7}{12}$. The shares of C and D together amount to £345. What does each receive?

2. A circle, 71 feet in diameter, is divided into two segments bearing the proportion of 32 : 81. Find the area of each segment. (The area of a circle is = ·7854 of that of its circumscribing square.)

3. A person set out at noon on a journey of 20 miles, walking at the rate of $3\frac{3}{4}$ miles an hour. On the way he looked at his watch, and found the time then passed from noon to be equal to $\frac{3}{5}$ of the time remaining till midnight. How far had he then to walk?

4. If 86 cwt. of coals be the amount of four loads, the second of which is equal to $\frac{5}{6}$ of the first, the third equal to $\frac{5}{8}$ of the second, and the fourth half as heavy again as the third, what is the weight of each load?

5. What fractional form equivalent to $\frac{5}{7}$ has the sum of its numerator and denominator 129?

6. What is the circumference of a circle when it is $7\frac{1}{2}$ feet longer than the diameter, reckoning the diameter of a circle to be equal to ·31831 of the circumference?

7. The hypotenuse of a right-angled triangle is 39 inches: find the base and perpendicular, 4 times the former being equal to 7 times the latter.

8. A is 24 years older than B. How old was each when A's age was equal to 5 times the half of B's?

9. Find the acreage of a rectangular field, the breadth of which is equal to $\frac{8}{15}$ of the length, and which a person, walking $3\frac{3}{11}$ miles an hour, can walk round in 5 minutes 45 seconds.

10. A man leaves his property to his wife, two sons, and three daughters, as follows:—His wife's share is to be double that of each of the sons, and three times that of each of the daughters; $2\frac{1}{2}$ per cent. has to be deducted to satisfy creditors, and the share of each daughter is then £1542 13*s.* 4*d.* What was the value of the property?

11. A's money exceeds B's by £12 7*s.* 11*d.*, and C's money is 3 per cent. less than A's, and 4 per cent. more than B's. What sum has each?

12. An acre of pasture consists of three fields: 8 times the extent of the first, 9 times that of the second, and 10 times that of the third would make equal areas. Find the extent of each field.

13. A certain sum is so divided among George, William, and Henry, that 5 times George's share is equal to 3 times William's, and twice William's equal to 7 times Henry's, and Henry's is 2*s*. 9*d*. less than George's. What sum is divided?

14. Divide £408 14*s*. into three such sums, that nine months' simple interest on the first at $3\frac{1}{2}$ per cent. per annum, on the second at $3\frac{3}{4}$ per cent., and on the third at $4\frac{1}{2}$ per cent., may be all equal.

15. Divide a square foot into three parts, so that $\frac{2}{3}$ of the first may be equal to $\frac{7}{8}$ of the second, and $\frac{5}{6}$ of the second equal to $\frac{4}{9}$ of the third.

16. Divide £2075 into two such sums, that if the first be put out at simple interest for 5 years at $3\frac{1}{2}$ per cent. per annum, and the second for 4 years at 3 per cent., the interest on the first sum shall be double that on the second.

17. The circumference of one circle is $27\frac{1}{2}$ feet longer than that of another, and 11 times the diameter of the first is equal to 5 times the circumference of the second. Find the diameter of each, reckoning the circumference of a circle equal to $3\frac{1}{7}$ times its diameter.

18. The perimeter of a quadrantal sector is 25 feet. Find its area, reckoning the circumference of a circle equal to the diameter multiplied by 3·1416, and the area equal to the square of the radius multiplied by 3·1416.

19. Divide £455 10*s*. into three parts, so that, by simple interest, the *amount* of the first for 4 months at $2\frac{1}{4}$ per cent. per annum, that of the second for 5 months at $3\frac{3}{4}$ per cent., and that of the third for 8 months at 6 per cent., shall be equal sums.

(*ii.*) *Irregular Distribution.*

4. We come now to exemplify what we have termed *Irregular Distribution* of quantities.

Ex. 1. In distributing half a guinea among James, Edward, and Harry, 18*d.* more was given to Edward than to James, and 3*d.* more to Harry than to Edward. How much did each receive?

In working a question of this kind take unity, or a convenient number of units, to represent the share of one of the persons, and from this derive a corresponding expression for each additional share. The above question, then, may be worked as follows:—

The half-guinea is equal to

1 unit for James's share,
+1 such unit + 18*d.* for Edward's „
+1 such unit + 18*d.* + 3*d.* for Henry's „

that is, 3 units + 39*d.* make up the 10*s.* 6*d.*;
∴ 3 units = 10*s.* 6*d.* − 3*s.* 3*d.* = 7*s.* 3*d.*; or 1 unit = 2*s.* 5*d.*;
hence James gets 2*s.* 5*d.*, Edward 3*s.* 11*d.*, Harry 4*s.* 2*d.* *Ans.*

Otherwise.

The half-guinea is equal to

1 unit for Harry's share,
+1 such unit − 3*d.* for Edward's „
+1 such unit − 3*d.* − 18*d.* for James's „

that is, 3 units − 24*d.* make up the 10*s.* 6*d.*;
∴ 3 units = 10*s.* 6*d.* + 2*s.* = 12*s.* 6*d.*; or 1 unit = 4*s.* 2*d.*;
hence Harry gets 4*s.* 2*d.*, Edward 3*s.* 11*d.*, James 2*s.* 5*d.* *Ans.*

Ex. 2. Divide £32 7*s.* 10*d.* among A, B, C, so that B's share may be 9*d.* more than one-half of A's share, and C's 7*d.* more than one-third of B's share.

The whole sum is equal to

1 unit for A's share,
$+\frac{1}{2}$ of that unit $+9d.$ for B's „
$+\frac{1}{6}$ of that unit $+3d.+7d.$ for C's „

that is, $1\frac{2}{3}$ of a unit $+1s.\ 7d. = £32\ 7s.\ 10d.$
$\therefore 1\frac{2}{3}$ of a unit $= £32\ 7s.\ 10d. - 1s.\ 7d. = £32\ 6s.\ 3d.$
hence 1 unit $= \frac{3}{5}$ of $£32\ 6s.\ 3d. = £19\ 7s.\ 9d.$ to A,
$\frac{1}{2}$ of £19 7s. 9d. + 9d. = 9 14 $7\frac{1}{2}$ to B,
$\frac{1}{6}$ of 19 7 9 +10 = 3 5 $5\frac{1}{2}$ to C. } *Ans.*

Otherwise.

To avoid fractions, let us regard the whole sum as equal to

6 units for A's share,
+3 units $+9d.$ for B's „
+1 unit $+3d.+7d.$ for C's „

that is, 10 units $+1s.\ 7d.$ make up the $£32\ 7s.\ 10d.$
$\therefore$ 10 units $= £32\ 7s.\ 10d. - 1s.\ 7d. = £32\ 6s.\ 3d.$
hence 1 unit $= £32\ 6s.\ 3d. \div 10 = £3\ 4s.\ 7\frac{1}{2}d.$;
$£3\ 4s.\ 7\frac{1}{2}d. \times 6 = £19\ 7s.\ 9d.$ to A,
$9d. + 3\ 4\ 7\frac{1}{2} \times 3 = 9\ 14\ 7\frac{1}{2}$ to B,
$3\ 4\ 7\frac{1}{2} + 10d. = 3\ 5\ 5\frac{1}{2}$ to C. } *Ans.*

Ex. 3. Divide $£54\ 1s.\ 6d.$ into three parts, so that the first shall be equal to $\frac{5}{7}$ of the second, and the third equal to $10s.\ 6d.$ less than the first and second together.

If we take 5 units to represent the 1*st* part,
we have 7 units for the 2*nd* „
and 12 units $- 10s.\ 6d.$ for the 3*rd* „ ;
that is, 24 units $- 10s.\ 6d. = £54\ 1s.\ 6d.$
$\therefore$ 24 units $= £54\ 12s.$; and 1 unit $= £2\ 5s.\ 6d.$;
hence 5 units $= £11\ 7s.\ 6d.$
7 units $= 15\ 18\ 6$
12 units $- 10s.\ 6d. = 26\ 15\ 6$ } *Ans.*

Ex. 4. Divide the number 238 into four such parts

that the first increased by 5, the second diminished by 7, the third multiplied by 9, and the fourth divided by 11, may give one common result.

Call the common result R; then,
the 1*st* part is equal to R−5,
2*nd* „ „ R+7,
3*rd* „ „ $\frac{1}{9}$R,
4*th* „ „ 11R;

$\therefore 238 = 13\frac{1}{9}R + 2$; or $118\,R = 236 \times 9$;
hence R = 18; and the parts are 13, 25, 2, and 198. *Ans.*

Ex. 5. What is that vulgar fraction which, if its numerator and denominator were each increased by 21, would become equal to $\frac{14}{27}$, but if they were each increased by 30 would become equal to $\frac{5}{9}$?

The fraction that would be found $= \frac{14}{27}$ would have a denominator $= \frac{14}{13}$ of the difference of its terms; and the fraction that would be found $= \frac{5}{9}$ would have a numerator $= \frac{5}{4}$ of the difference of its terms; but the difference in the original fraction and in the altered fractions is uniform, because numerator and denominator were increased *equally*; hence, calling the uniform difference D, we have

$\frac{5}{4}$D = the orig. numerator + 30,
and $\frac{14}{13}$D = the orig. numerator + 21;
$\therefore \frac{9}{52}$D = 9, and hence D = 52;
$\frac{14}{13}$ of 52 = 56, and $\frac{56-21}{108-21} = \frac{35}{87}$. *Ans.*

EXERCISES 6.

1. Divide 7 guineas among A, B, C, giving B 3*s.* more than A, and C thrice as much as B.

2. Divide £19 among A, B, C, D, giving B 5*s.* less than A, C 10*s.* less than B, and D as much as the other three together.

3. 23½ acres of land are divided into three parts; the first contains 5 acres for every 14 roods in the second, and the third 2 acres less than the first and second together; what is the extent of each part?

4. Divide £6 among A, B, C, D, making B's share equal to $\frac{2}{3}$ of A's, C's equal to $\frac{4}{5}$ of B's, and D's 4*s.* more than the sum of A's and B's.

5. Divide the number 80 into four such parts, that the first increased by 3, the second diminished by 3, the third multiplied by 3, and the fourth divided by 3, may all give the same result.

6. Divide a square pole into four such parts that the first increased by 3 square yards, the second diminished by 2¼ square yards, the third multiplied by 6, and the fourth divided by 3, will give equal results.

7. Find two sums of money such that the first when diminished by 12 per cent. shall be less than the second by £10 12*s.*, and the second when increased by 4 per cent. shall be less than the first by £10 12*s.*

8. Divide 44*s.* 2*d.* among A, B, C, giving B 7*s.* more than A, and C 7 per cent. more than B.

9. Find that vulgar fraction which, if its terms be each increased by 1, is made equal to $\frac{5}{11}$, but if they are each diminished by 1 becomes equal to $\frac{1}{3}$.

CHAPTER VI

PROBLEMS RELATING TO THE RATES OF TWO TRAVELLERS, ADVANCING TO MEET EACH OTHER, OR ONE FOLLOWING TO OVERTAKE THE OTHER.

Ex. 1. M and N are two stations 30 miles apart; A sets out from M towards N, and rides at the rate of $10\frac{1}{2}$ miles an hour; and B at the same time from N towards M, riding $7\frac{1}{2}$ miles an hour. When, and how far from M, will they meet?

Here we have A and B covering $10\frac{1}{2}+7\frac{1}{2}=18$ miles of the road between them per hour; hence they will cover the whole of the road between them, and therefore will meet, in $30\div18=1\frac{2}{3}$ hour, or 1 hour 40 minutes. *1st Ans.*

A at that time will be $10\frac{1}{2}$ miles $\times 1\frac{2}{3}$, or $3\frac{1}{2}$ miles $\times 5$ $=17\frac{1}{2}$ miles from M. *2nd Ans.*

Ex. 2. The distance between two places, M and N, is 17 miles. A sets out from M towards N at the rate of $3\frac{3}{4}$ miles an hour, and B, 40 minutes later, from N towards M at $3\frac{1}{2}$ miles an hour. How long and how far will B have travelled when he meets A?

A has travelled $\frac{2}{3}$ of an hour, at $3\frac{3}{4}$ miles per hour, $=2\frac{1}{2}$ miles, when B starts, and then there are $17-2\frac{1}{2}$ or $14\frac{1}{2}$ miles to be covered between them, at the combined rates of $3\frac{3}{4}$ and $3\frac{1}{2}$ miles$=7\frac{1}{4}$ miles per hour; hence $14\frac{1}{2}\div7\frac{1}{4}=2$ hours' travel by B $=7$ miles. *Ans.*

Ex. 3. L sets out from A towards B, and M at the same time from B towards A, a distance of 94 miles. They meet in 8 hours, L's rate per hour having been $2\frac{3}{4}$ miles more than M's. Find the rate of each.

As the distance was gone between them in 8 hours, the sum of their rates must have been $94 \div 8 = 11\frac{3}{4}$ miles an hour, and the difference of their rates is $2\frac{3}{4}$ miles; hence L's rate is $\frac{1}{2}(11\frac{3}{4} + 2\frac{3}{4}) = 7\frac{1}{4}$ miles, and M's $4\frac{1}{2}$. *Ans.*

Ex. 4. Find the exact time between 5 and 6 o'clock when the minute hand of a clock is one-sixth of the circumference of the dial in advance of the hour hand.

Call the minute hand M, and the hour hand H. These are, as it were, two travellers, M following H in a ring 60 minute spaces round. Now, H passes over 5 spaces in an hour, while M passes over 60 spaces; so that M gains on H 55 spaces per hour. At 5 o'clock H is 25 spaces in advance of M, so that the latter has $25 + 10 = 35$ spaces to gain in order to get 10 spaces beyond H; if, therefore, 55 spaces be gained in an hour, 35 spaces will be gained in $\frac{35}{55}$ or $\frac{7}{11}$ of an hour; hence 420 minutes $\div 11 = 38\frac{2}{11}$ minutes past 5. *Ans.*

Note.—If the question had proposed to find the time at which there would be an interval of 10 minute spaces between the hands, another answer besides the above would have been wanted; for that interval would also occur when M gained $25 - 10$ or 15 spaces, viz. in $\frac{15}{55}$ or $\frac{3}{11}$ of an hour. Compare next example.

Ex. 5. At what time between 6 and 7 o'clock will the hour and minute hands of a watch intercept an eighth part of the circumference of the dial?

At 6 o'clock M is 30 minute spaces behind H; but as M gains on H at the rate of 55 spaces per hour, it

will be only $7\frac{1}{2}$ spaces behind H in $\frac{22\frac{1}{2}}{55}$ or $\frac{9}{22}$ of an hour, viz. at 540 min. $\div 22 = 24\frac{6}{11}$ minutes past 6. *1st Ans.*

Also, M will be $7\frac{1}{2}$ spaces in advance when it has gained $37\frac{1}{2}$ spaces, viz. $\frac{37\frac{1}{2}}{55}$ or $\frac{15}{22}$ of an hour; that is, at 900 min. $\div 22 = 40\frac{10}{11}$ minutes past 6. *2nd Ans.*

Ex. 6. Three athletes run a 3-mile race round a course of 528 yards, their rates being as 32 : 30 : 22. Find the positions of the other two when the winner comes in.

3 miles being 5280 yards, the first will, of course, have run 10 rounds; then, the rates being as 1, $\frac{15}{16}$, and $\frac{11}{16}$, the second will have run $\frac{15}{16}$ of 10 rounds$=9\frac{3}{8}$ rounds, and the third will have run $\frac{11}{16}$ of $10 = 6\frac{7}{8}$ rounds.

Hence the third runner will be $\frac{1}{8}$ of a round, or 66 yards, behind the winner; and the second will be $\frac{5}{8}$ of a round, or 330 yards, behind the winner. *Ans.*

Ex. 7. A sets out from M to go to another place N, and at the same time B sets out from N for M, B's rate being equal to $\frac{3}{4}$ of A's. They meet in 8 hours 40 minutes. How long does each person take to perform his whole journey?

Here A goes 4 units of distance per hour, and B 3 such units; 7 units together per hour; therefore the whole distance is $7 \times 8\frac{2}{3} = 60\frac{2}{3}$ units. Accordingly, for that distance A takes $60\frac{2}{3} \div 4 = 15\frac{1}{6}$ hours, and B $60\frac{2}{3} \div 3 = 20\frac{2}{3}$ hours. *Ans.*

Ex. 8. A is rowing down a river to meet B rowing up. They are 17 miles apart. A's power of rowing in still water is $5\frac{1}{8}$ miles, and B's $5\frac{1}{2}$ miles, an hour; and,

when they meet, A has rowed $3\frac{2}{5}$ miles more than B. At what rate does the river flow?

Here, as by the force of the current one man's power is augmented just as much as the other's is reduced, the sum of their actual rates is $5\frac{1}{8}+5\frac{1}{2}=10\frac{5}{8}$ miles, and therefore they meet in $17\div10\frac{5}{8}=1\frac{3}{5}$ hours; hence the difference of their actual rates per hour is $3\frac{2}{5}\div1\frac{3}{5}$ $=2\frac{1}{8}$ miles = twice the rate of the stream $-(5\frac{1}{2}-5\frac{1}{8})$; accordingly, $(2\frac{1}{8}+\frac{3}{8})\div2=1\frac{1}{4}$ miles an hour. *Ans.*

Ex. 9. A and B run a 4-mile race; and when A has run $2\frac{1}{2}$ miles, B has run $2\frac{3}{8}$. In what ratio must B increase his speed from this point, in order to win by 300 yards?

300 yards is $\frac{15}{88}$ of a mile. A is $1\frac{1}{2}$ miles from the end, B is $1\frac{5}{8}$ miles from it. B, therefore, must run the $1\frac{5}{8}$ miles in the time A will have run $1\frac{1}{2}-\frac{15}{88}=1\frac{29}{88}$; and these distances are as $13\times11:117$, or as $11:9$. Accordingly, as B's rate at first was $\frac{9}{10}$ of A's, and must now be $\frac{1}{9}$ of A's, he must increase his speed in the ratio of $\frac{1}{9}$ to $\frac{9}{10}$ or 220 to 171. *Ans.*

Ex. 10. X, Y, and Z travel from the same place in one direction, X's rate per hour being $2\frac{3}{4}$ miles, Y's $3\frac{1}{2}$ miles, and Z's $4\frac{1}{8}$. If Y starts 2 hours after X, how long after Y must Z start, that they may both overtake X at the same time?

X in two hours goes $5\frac{1}{2}$ miles; and as Y gains this at the rate of $3\frac{1}{2}-2\frac{3}{4}$ or $\frac{3}{4}$ of a mile per hour, he overtakes X in $7\frac{1}{3}$ hours, and has then travelled $3\frac{1}{2}$ mi. $\times7\frac{1}{3}$ $=25\frac{2}{3}$ miles. Z can go that distance in $25\frac{2}{3}\div4\frac{1}{8}$ $=77\times8\div33\times3=56\div9=6\frac{2}{9}$ hours; $\therefore$ Z should start $7\frac{1}{3}-6\frac{2}{9}=1\frac{1}{9}$ hour after Y. *Ans.*

Ex. 11. On the road to Brighton A follows B, who is

9 miles ahead of him. A rides $8\frac{3}{4}$ miles an hour, and B $6\frac{1}{4}$, and B will reach Brighton just 6 minutes before A. How far is B from Brighton?

Call the required distance d.

In 6 minutes A would ride $\frac{1}{10}$ of $8\frac{3}{4} = \frac{7}{8}$ of a mile; $\therefore$ he travels $9 + d - \frac{7}{8}$ or $8\frac{1}{8} + d$ miles while B travels d miles; thus A, by riding $2\frac{1}{2}$ miles more per hour than B, gains on him $8\frac{1}{8}$ miles, which is done in $8\frac{1}{8} \div 2\frac{1}{2}$, or $3\frac{1}{4}$ hours; so that B's distance from Brighton is $6\frac{1}{4}$ miles $\times 3\frac{1}{4} = 20\frac{5}{16}$ miles. *Ans.*

Ex. 12. L and M set out together from the same place, and in the same direction. L travels uniformly 14 miles a day, and after 6 days turns and goes back as many miles as M had travelled forward during those 6 days. He then returns and, pursuing his journey, overtakes M in 16 days from their first setting out. How many miles a day had M travelled?

The whole number of miles travelled by L is equal to the whole number by M + the number travelled by M in $6 + 6$ days; that is, $14 \times 16 =$ M's rate $\times (16 + 12)$ days; $\therefore$ M's rate $= 14$ miles $\times 16 \div 28 = 8$ miles a day. *Ans.*

Ex. 13. A person performs a journey at a certain rate. If he had travelled $1\frac{1}{2}$ mile an hour faster, he would have accomplished the journey in $\frac{7}{10}$ of the time; whereas, if he had travelled a mile per hour slower, he would have been two hours longer on the road. Find the whole distance travelled.

$1\frac{1}{2}$ mile per hour faster was $\frac{10}{7}$ of his rate; $\therefore$ his rate was $1\frac{1}{2}$ mi. $\div \frac{3}{7} = 3\frac{1}{2}$ miles an hour; and hence, at $2\frac{1}{2}$ miles an hour he would have taken $\frac{3\frac{1}{2}}{2\frac{1}{2}}$ or $\frac{7}{5}$ of the time; $\therefore$ the 2 hours longer $= \frac{2}{5}$ of the time, or the

time was 5 hours, and hence the whole distance was $3\frac{1}{2}$ miles $\times 5 = 17\frac{1}{2}$ miles. *Ans.*

Ex. 14. L, M, and N start together from one point in a ring fence 880 yards in compass, and walk round it in one direction. L goes 80 yards per minute, M 110 yards, N 120. How soon will they all come together again? and what time would be required to bring them together again at the starting-point?

In 1 minute L goes $\frac{1}{11}$ of a round, M $\frac{1}{8}$, N $\frac{3}{22}$; $\therefore$ M gains on L $\frac{3}{88}$ of a round per minute, and N gains on L $\frac{1}{22}$ of a round per minute; so that M would gain on L a whole round in $88 \div 3$ or $29\frac{1}{3}$ minutes, and N a whole round in 22 minutes; hence M and N will first come together with L in the time which is the least common multiple of $29\frac{1}{3}$ and 22 minutes, viz. in 88 minutes, or 1 hour 28 minutes. *1st Ans.*

Again, as a complete round is walked by L in 11 minutes, by M in 8 minutes, and by N in $7\frac{1}{3}$ minutes, the three would come together again at the starting-point in the time which is the least common multiple of these times, viz. in 88 minutes, or 1 hour 28 minutes; whence it appears that, at the given rates of walking, they could never be together at any other point than that from which they started. *2nd Ans.*

EXERCISES 7.

1. A is 24 miles ahead of B, who follows him. A is going 5 miles an hour and B 7; but after 4 hours A's rate is increased to 6 and B's to 9 miles an hour. How soon after this will B overtake A?

2. If a train 90 yards long, going 40 miles an hour, meets another train going 35 miles an hour, and they pass each other in 4·8 seconds, what is the length of

the other train? And how long would the faster train have taken to pass the slower, if they had been going in the same direction?

3. A and B are to cut a uniform trench from P to Q, two stations 322 yards apart. A begins at P, and B, $1\frac{1}{2}$ hour later, at Q, and they work towards each other. At A's rate of working a portion of the trench, $2\frac{1}{2}$ yards long, is formed in an hour; at B's rate a portion, $2\frac{1}{9}$ yards long, is done in the hour. In what time is the trench completed?

4. At what time between 10 and 11 o'clock will the hour and minute hands of a watch be at right angles to each other? Also, when will they be coincident? and when will they point in opposite directions?

5. Two bicyclists, L and M, set out from one place and ride in the same direction. L rides $8\frac{1}{4}$ miles an hour, and M, who starts 25 minutes later than L, rides $10\frac{3}{4}$ miles an hour. How far will each have travelled when M overtakes L?

6. A travels from Edinburgh to Glasgow, and B from Glasgow to Edinburgh. Both start at the same time, A's rate per hour being equal to $\frac{2}{3}$ of B's; and they meet in 4 hours. In what time does each travel the whole distance?

7. A and B walked from M to N, and C from N to M, a distance of $25\frac{1}{3}$ miles. A's rate per hour was $3\frac{1}{3}$ miles, B's $3\frac{3}{4}$, C's $4\frac{1}{6}$. B left M when C left N, but A had started some time before them. How long must that have been, if A was overtaken by B just at the moment when they were met by C?

8. A rides at the rate of 10 miles an hour from M towards N to meet B, who starts $13\frac{1}{5}$ minutes later than A, from N towards M, riding at the rate of $11\frac{1}{9}$

miles an hour. When they meet, each has travelled the same number of miles. Find that number.

9. A, B, and C start together at one point in a circular course, and go round it in one direction. A completes a round in $9\frac{1}{2}$ minutes, B in 5 minutes 42 seconds, C in $4\frac{1}{2}$ minutes. In what time should they be all together again at the starting-point? and in what time should they first come together?

10. A and B start from the same point to run in opposite directions round a circular race-course, 9755 feet in circumference, A not starting until B has run 105 feet. They pass each other when A has run 4850 feet. Which will first come round again to the starting-point (their speeds being uniform throughout), and what distance will they then be apart?

11. A and B engage to walk a match of 10 miles round a course of 5 furlongs. A during the first eight rounds gains 110 yards each round upon B; and then, their rates being reversed, B gains similarly on A at the rate of 110 yards each round for the rest of the race. Which comes in first, and by how many yards does he win?

12. A, B, and C walked from P to Q, each at a uniform rate, A's rate being equal to $\frac{4}{5}$ of C's, and B's rate 4 miles an hour. B started 45 minutes later than A, and C started 27 minutes later than B, and all arrived at Q at the same time. Find the distance of Q from P.

13. A and B set out at the same time from P and Q respectively, and each rode at a uniform rate towards the original station of the other. B, riding $8\frac{1}{4}$ miles an hour, met A in 2 hrs. 40 min., and in 2 hrs. 56 min. after that A arrived at Q. Find A's rate, and the distance from P to Q.

CHAPTER VII.

DIFFERENT WAYS OF INDICATING GAIN AND LOSS.

FOR many indirect ways of indicating gain or loss no general rule can be given. They should be studied by young pupils in worked examples, such as are here supplied.

Ex. 1. A grocer sells goods at the rate of 8 lbs. for 9 shillings, which he bought at the rate of 9 lbs. for 8 shillings; what does he gain per cent.?

He buys at $\frac{8}{9}$ of a shilling per lb. and sells at $\frac{9}{8}$ of a shilling per lb. Hence, prime cost is to selling price as $\frac{8}{9} : \frac{9}{8}$, or as 64 : 81. If, therefore, 17 be gained on 64, the gain on 100 is $\frac{100}{64}$ of $17 = 26\frac{9}{16}$ per cent. *Ans.*

Ex. 2. If 174 lbs. 6 oz. avoirdupois be bought for £52 11*s.* 5*d.*, and 12 per cent. of the whole be allowed for waste, at how much per lb. must the remainder be sold to gain $12\frac{1}{2}$ per cent.?

$\frac{88}{100}$ of $174\frac{3}{8}$ lbs. is to be sold for $\frac{112\frac{1}{2}}{100}$ of £52 11*s.* 5*d.*;

or 11 times 1395 lbs. to be sold for 225 times £26 5*s.* $8\frac{1}{2}d.$,

or 11 times 31 lbs. to be sold for 5 times £26 5*s.* $8\frac{1}{2}d.$,

or 31 lbs. for £2 7*s.* $9\frac{1}{2}d. \times 5$; or 1 lb. for 1*s.* $6\frac{1}{2}d. \times 5$;

$= 7s.\ 8\frac{1}{2}d.$ *Ans.*

Ex. 3. A corn factor having bought 2400 quarters of barley, sold $\frac{3}{5}$ of it at a profit of $12\frac{1}{2}$ per cent., $\frac{1}{4}$ at

a profit of 15 per cent., and the rest at cost price. Had he sold all the barley at a profit of 10 per cent. he would have realised £47 5*s.* less. Find the cost of the barley.

Here the quantity 2400 quarters is not wanted in the calculation.

$\frac{3}{5}$ sold at $\frac{112\frac{1}{2}}{100}$ of cost price $= \frac{27}{40}$ of cost price;

$\frac{1}{4}$ „ $\frac{115}{100}$ „ $= \frac{23}{80}$ „

$\frac{3}{20}$ „ $\frac{100}{100}$ „ $= \frac{3}{20}$ „

The whole was sold for $\frac{89}{80}$ of its cost;

$\therefore \frac{89}{80} - \frac{110}{100}$, or $\frac{1}{80}$ of the cost, was £47$\frac{1}{4}$;

hence the whole cost was £189 × 20 = £3780. *Ans.*

Ex. 4. A tradesman who makes 10 per cent. on his capital allows 2*d.* in the shilling discount off the price marked on each article. How much per cent. above cost price does he mark the articles for sale?

What costs 100 is sold for 110, the marked price for which must be $\frac{12}{10}$ of 110, or 132 = 32 per cent. above cost price. *Ans.*

Ex. 5. What is the difference between a loss of 5 per cent. on the prime cost and on the selling price of an article sold for £56?

When 100 is the prime cost, the loss is 5, and the selling price 95; thus a loss of 5 per cent. on prime cost is always $\frac{1}{19}$ of selling price; so that the difference here required is equal to $(\frac{1}{19} - \frac{1}{20})$ of £56 = $\frac{1}{19}$ of 56*s.* = 2*s.* $11\frac{7}{19}$*d.* *Ans.*

Ex. 6. A person bought an article, and sells it so as to gain 5 per cent. If he had bought it for 5 per cent. less, and sold it for 1 shilling less, he would have gained 10 per cent. Find the cost price.

For an outlay of 100 the return is 105;
for an outlay of 95 the return would have been $\frac{110}{100}$ of $95 = 104\frac{1}{2}$;
$\frac{105-104\frac{1}{2}}{100}$ of prime cost, or $\frac{1}{200}$ of prime cost is 1*s.*;
hence the prime cost $= 200s. = £10$. *Ans.*

Ex. 7. I bought goods at 23*s.* 9*d.* with 4 months' credit, and sold them forthwith at 25*s.* 6*d.*, with such allowance of credit as made my immediate gain $6\frac{2}{3}$ per cent. How long credit did I give, reckoning interest at 4 per cent. per annum?

Present worth of 23*s.* 9*d.* for $\frac{1}{3}$ of a year at 4 per cent. per annum $= \frac{100}{101\frac{1}{3}}$ of $\frac{95s.}{4} = \frac{75}{76}$ of $\frac{95s.}{4} = \frac{375s.}{16}$;
and the amount of this at $6\frac{2}{3}$ per cent. profit will be $= \frac{106\frac{2}{3}}{100}$ of $\frac{375s.}{16} = \frac{375s.}{15} = 25s.$, which therefore is the present worth of 25*s.* 6*d.* for the time sought.

We have, accordingly, to find the time for which 6*d.* is the interest on 25*s.* at 4 per cent. per annum. Now, the interest for 12 months is 1*s.*; hence 6*d.* is the interest for 6 months. *Ans.*

Ex. 8. At what price must a merchant tailor sell dress suits which cost him £3 2*s.* 6*d.* each, that his profit on £100 may be equal to the amount he will get for a dozen suits?

$£100 \div £3\frac{1}{8} = 32$ suits. These cost him £100;
hence, he must sell $32 - 12 = 20$ suits for £100,
that is, at £5 the suit. *Ans.*

EXERCISES 8.

1. A tobacconist sells 25 cigars for the prime cost of 28; what does he gain per cent.?

2. What profit does a fruiterer make on 3 dozen lemons, if in buying he gives 13*d.* for 16 lemons, and in selling gives a dozen lemons for 17*d.*?

3. A person by selling, at 2*s.* 9¾*d.* per lb., an article which cost £14 per cwt., makes 5 per cent. more profit than he would make if he sold the whole for £55 15*s.* 3¾*d.* What was the quantity sold?

4. A merchant buys 3150 yards of cloth. He sells ⅓ of it at a gain of 6 per cent., ⅕ at a gain of 8 per cent., ⅐ at a gain of 12 per cent., and the rest at a loss of 3 per cent. Had he sold the whole at a gain of 5 per cent., he would have received £12 1*s.* 6*d.* more than he did. What was the prime cost of 1 yard?

5. Bought brandy at 19*s.* 2*d.* a gallon; at what price must I sell it that ⅐ of the selling price may be profit?

6. A man embarks his whole property in four successive ventures. In the first he clears 100 per cent., and in each of the others he loses 20 per cent. Show that there remains to him 2⅔ per cent. profit on his original outlay.

7. When 8¼ per cent. of what goods cost me is equal to 6⅔ per cent. of what I sell them for, what do I gain per cent.?

8. If a merchant sells 15 tons of coal for the prime cost of 17¼ tons, how much per cent. of the selling price is his profit?

9. I bought a flock of sheep for a certain sum, the

number of sheep being the same as the number of shillings I paid for each. I lost a tenth part of them; but by selling the rest at an advance of 20 per cent. on the cost price per head, I gained £3 12*s*. How many sheep did I buy?

10. A, having bought a quantity of goods, sold them at an advance of $2\frac{1}{2}$ per cent. to B, who sold them at a loss of $1\frac{1}{2}$ per cent. to C; but C made $1\frac{1}{4}$ per cent. profit by selling them to D, who disposed of them at a loss of 2 per cent., for a sum equal to 18*s*. more than A had given for them. What did the goods cost A?

11. If cigars cost a merchant 26*s*. 3*d*. a hundred, at what must he sell them per bundle of 60, to gain the selling price of 25 dozen on an outlay of 30 guineas?

12. If cheese be bought at 74*s*. a cwt., how must it be sold to gain $12\frac{1}{2}$ per cent., after an allowance of $7\frac{1}{2}$ per cent. discount?

13. Bought tea at 2*s*. 8*d*. a lb. ready money, and sold it the same day at 3*s*., allowing 3 months' credit. What was my immediate gain per cent., interest being reckoned at 5 per cent.?

14. Bought a quantity of corn at 30*s*. with 3 months' credit; how must I now sell it to gain $7\frac{1}{2}$ per cent., giving 9 months' credit, reckoning interest at 5 per cent.?

15. M offers to buy of me a quantity of wine at 9*s*. 9*d*. a gallon, cash, but I should lose $2\frac{1}{2}$ per cent. by accepting that offer. He agrees, however, to give me a credit price of 11*s*. a gallon, which makes my present gain $9\frac{1}{11}$ per cent., interest being rated at 5 per cent. How long credit do I allow?

CHAPTER VIII.

VARIETIES OF FORM IN PROBLEMS ON STOCKS.

THE student being here presumed to have an elementary knowledge of the general nature of transactions in stocks, we have endeavoured to devise means of simplifying the consideration of various indirect forms in which problems in this department of Arithmetic are often proposed, and which prove more or less perplexing to many that are fairly up in most other Rules.

One thing very necessary to be attended to is the distinction between stock money at the owner's credit, and tangible cash invested in the purchase of that amount of credit, or obtained by disposing of it—selling out, as it is called.

Suppose I invest £2320 in the 3 per cents. at 87, the meaning is that I pay at the rate of £87 cash for every cent. of stock, that is, for every £100 of stock, and the number of times that my payment, £2320, contains £87 will be the number of cents. of stock which I shall obtain. Now $2320 \div 87 = 26\frac{2}{3}$, and $26\frac{2}{3}$ times £100 = £$2666\frac{2}{3}$, the amount of stock. Be careful not to divide stock by cash, or cash by stock, in seeking to ascertain the number of cents. owned. The amount of cash paid, £2320, divided by the cash rate paid for a cent., viz. £87, will give the number of cents., viz. $26\frac{2}{3}$; or the amount of stock bought, viz.

£2666⅔, divided by the value of a cent. in stock money, viz. £100, will give the same number of cents., 26⅔.

So also to find the annual income obtained by investment, as £3 is the income for every £87 of cash paid, or for every £100 of stock bought, we have either $\frac{£2666\frac{2}{3} \times 3}{100}$ or $\frac{£2320 \times 3}{87} = £80$ a year.

Notice, then, generally, that when a person is said to have a sum in any stock, that sum is stock, not cash; and when he is said to invest a sum in any stock, that sum is cash, not stock. We proceed to more particular illustrations by means of problems.

Ex. 1. A gentleman invests £7600 in the 3½ per cents. when they are at 83⅛, and on the price rising to 87⅞ he sells out, and invests the proceeds in 5 per cent. stock at 114. Find the alteration in his income.

Here we have to find the difference between his first income and his second.

To find the first income, we seek to know how many cents. of stock he has at first, then his income will be £3½ for every cent. Now he paid £83⅛ for each cent., and £7600 for all his cents.; he therefore purchased

$$\frac{7600}{83\frac{1}{8}} = \frac{7600 \times 8}{665} = \frac{400 \times 8}{35} = \frac{640}{7} \text{ cents};$$

which calculated at £3½ the cent. = £320, his first income.

Again, he sells the cents. at 87⅞, by which he gets $£87\frac{7}{8} \times \frac{640}{7}$ *cash*; and as often as this amount of cash contains £114 he will obtain so many cents. in the other fund; we therefore divide the amount of cash by £114, and for each cent. his income will be £5; thus,

$$\frac{£5 \times 87\frac{7}{8} \times \frac{640}{7}}{114} = \frac{£5 \times 703 \times 80}{114 \times 7} = \frac{£5 \times 37 \times 40}{3 \times 7}$$

$= £7400 \div 21 = £352\frac{8}{21}$, or an increase of $£32\frac{8}{21}$. *Ans.*

Ex. 2. A person invests £1060 in $3\frac{1}{4}$ per cent. stock when it is at $86\frac{1}{8}$; afterwards, when the stock is at 91, he sells out and invests the proceeds in a mortgage paying him 5 per cent. What alteration does this transaction make in his income?

No. of cents. at first $= \frac{1060}{86\frac{1}{8}} = \frac{1060 \times 8}{689} = \frac{20 \times 8}{13}$,

which at £$3\frac{1}{4}$ income for each = £40, his 1*st* income.

Again, he sells the cents. at 91, by which he obtains $\frac{£91 \times 160}{13}$ cash; and with this amount he then buys as many times £5 of income as the amount contains £100; all which may be expressed by $\frac{£91 \times 160 \times 5}{100 \times 13}$

= £7 × 8 = £56; his 2*nd* income, showing an increase of £16. *Ans.*

Ex. 3. A man whose income is derived from £40000 stock in the 3 per cent. Consols sells out at 95, and invests half the proceeds of the sale in India 4 per cents. at 104, and the other half in Canadian Railways, paying 4 per cent., at $108\frac{1}{2}$. Find, within a penny, how his income is affected.

£40000 ÷ £100 = 400 cents.; so his 1*st* income is = £3 × 400, or £1200.

£95 × 400 = the whole amount of the sale in cash.

With £95 × 200 cash he buys an income, at the rate of £4 of income for £104 cash; the amount of that income, therefore, is = $\frac{1}{104}$ of £4 × 95 × 200.

With the remaining £95 × 200 he buys an income at the rate of £4 income for $£108\frac{1}{2}$ cash; the amount of this income is accordingly = $\frac{1}{108\frac{1}{2}}$ of £4 × 95 × 200.

∴ whole income = £4 × 95 × 200 × $(\frac{1}{104} + \frac{2}{217})$

$= £380 \times 200 \times \frac{425}{104 \times 217} = \frac{£19000 \times 1700}{22568}$

= £1431 4*s.* 7*d.*, his 2*nd* income, being an increase of £231 4*s.* 7*d.* *Ans.*

Ex. 4. A gentleman invests £2500 in the 4 per cents. at $101\frac{1}{4}$. What interest has he for his money?

The question tells that he gets £4 per annum on the outlay of $£101\frac{1}{4}$; and we can see, without taking the £2500 into calculation, that the rate of interest per cent. per annum is $= \frac{100}{101\frac{1}{4}}$ of $£4 = \frac{1600}{405} = 320 \div 81$ $= 3\frac{77}{81}$ per cent. *Ans.*

Ex. 5. What must be the market value of $3\frac{1}{2}$ per cent. stock, that, after deducting income-tax of 5*d.* in the pound, it may yield $3\frac{11}{12}$ per cent. interest on investments?

By deducting an income tax of 5*d.* in the pound, the net income is $= \frac{235}{240}$ of the gross income. So the question is, to find what cash investment will obtain an income of $£3\frac{1}{2} \times \frac{235}{240}$, when an investment of £100 would obtain $£3\frac{11}{12}$.

$\frac{7}{2} \times \frac{47}{48} \times \frac{12}{47}$ of $100 = \frac{7}{8}$ of $100 = 87\frac{1}{2}$. *Ans.*

Ex. 6. How much 3 per cent. stock must be sold out at $89\frac{1}{8}$, in order that the owner's income may be made £12 better by investing the proceeds in 4 per cent. stock at $91\frac{7}{8}$? The brokerage on each sale is $\frac{1}{8}$ per cent.

By reason of the charge for brokerage, the owner will get only £89 for each cent. he sells, and must pay £92 for each cent he buys.

The 1*st* income is = £3 for each cent. of the stock to be sold.

By selling out, £89 cash will be got for each cent., and by buying in, £4 income will be got for every £92

contained in that cash, or this income will be $\frac{89}{92}$ of £4 for every cent. sold out; hence the 1*st* income is $= \frac{3}{100}$ of the stock sold, and the 2*nd* income is $= \frac{89}{92}$ of $\frac{4}{100}$ of the stock sold.

Hence the difference $= \frac{89}{92}$ of $\frac{4}{100}$ and $\frac{3}{100}$ of the stock sold $= \frac{89}{2300} - \frac{69}{2300} = \frac{1}{115}$ of the stock = £12; and, therefore, the stock sold = £12 × 115 = £1380. *Ans.*

Ex. 7. A man transfers £720 of $3\frac{1}{2}$ per cent. stock to the 4 per cents. at par, and thus makes his income 12 per cent. better. At what price did he sell out from the $3\frac{1}{2}$ per cents.?

The price of the 4 per cents. is £100 cash for £100 stock, so that the income from the 4 per cents. is equal to £4 for every £100 invested; and as this is 12 per cent. better than the 1*st* income, we have the value of the 1*st* income $= \frac{100}{112}$ of $\frac{4}{100} = \frac{1}{28} =$ £1 for an investment of £28 = £$3\frac{1}{2}$ for an investment of £$3\frac{1}{2} \times 28$, or 98, the price required. *Ans.*

Ex. 8. By buying 3 per cent. Consols at a certain price, I find I obtain $3\frac{3}{5}$ per cent. interest for my money, and derive therefrom, after paying an income-tax of 6*d.* in the pound, a net income of £421 4*s.* Find the amount of stock, and the price at which I bought it.

Gross income $= \frac{40}{39}$ of £$421\frac{1}{5}$ = £432.

Now, as £432 of income is at the rate of £3 for every £100 of stock, the amount of stock is = £43200 ÷ 3, or £14400. 1*st Ans.*

Again, if an investment of £100 would obtain an income of $3\frac{3}{5}$, the investment that is the price for an income of 3 is equal to $\frac{3}{3\frac{3}{5}}$ or $\frac{5}{6}$ of 100 = $83\frac{1}{3}$. 2*nd Ans.*

Ex. 9. A gentleman has a sum of money invested in the 3 per cent. Consols. He sells out at $87\frac{1}{4}$, and

invests the proceeds in railway shares when a £100 share sells for £174½. By this transaction he increases his income from £120 to £200. What dividend per cent. per annum does the railway pay?

His income from the 3 per cents. having been £120, he must have had $120 \div 3 = 40$ cents., or £4000 stock.

He sells 40 cents. at 87¼, and with this cash he buys £100 railway shares at 174½. His railway property is $= \frac{£87\frac{1}{4} \times 40 \times 100}{174\frac{1}{2}} = £2000$; and as his income from this is £200, the railway pays a dividend of 10 per cent. *Ans.*

Ex. 10. A person transfers £1000 from the 3 per cents. at 90 to the 3½ per cents. at 102½. What alteration per cent. does he make in his income?

The incomes per pound of investment are $\frac{3}{90}$ and $\frac{7}{205}$ of £1, which are as 205 : 210, or 41 : 42; hence there is an increase per cent. $= \frac{1}{41}$ of $100 = 2\frac{18}{41}$. *Ans.*

Ex. 11. A gentleman invested £3585, part in 3½ per cent. stock at 95, and the remainder in 4 per cent. stock at 108; and his whole net income from these two sources, after deducting an income-tax of 6*d.* in the pound, was £129 3*s.* 9*d.* How much did he invest in each stock?

Gross income $= \frac{40}{39}$ of $£129\frac{3}{16} = £3\frac{5}{16} \times 40 = £132\frac{1}{2}$.

The two incomes together are $= \frac{3\frac{1}{2}}{95}$ of one of the parts invested $+ \frac{4}{108}$ of the other; or $\frac{7}{190}$ of one part $+ \frac{1}{27}$ of the other $= £132\frac{1}{2}$;

$\therefore \frac{189}{190}$ of one part + the other $= £3577\frac{1}{2}$;

but one part + the other $= £3585$;

$\therefore \frac{1}{190}$ of one part $= £7\frac{1}{2}$; or one part $= £1425$, and the other $= £2160$. } *Ans.*

Ex. 12. A man buys 3 per cent. stock at $89\frac{3}{8}$, and, after receiving a half-year's dividend, sells out at $94\frac{5}{8}$, and thus gains £81. What did he invest at first?

His gain of £81 was $= \frac{1\frac{1}{2}}{89\frac{3}{8}} + \frac{5\frac{1}{4}}{89\frac{3}{8}}$ of the investment,

$= \frac{12+42}{715}$ or $\frac{54}{715}$ of the investment; hence the investment $= £81 \div \frac{54}{715} = £1072\ 10s.$ *Ans.*

Ex. 13. I invested £3640 in the $4\frac{1}{2}$ per cents., and on the price falling $\frac{1}{2}$ per cent. I transferred the whole to a 6 per cent. stock at 92, and thereby augmented my income by £78 15*s.* At what price did I buy into the $4\frac{1}{2}$ per cents.?

Write N for the number of cents. bought at first;

$3640 - \frac{1}{2}N = \frac{92}{6}$ of 2*nd* income,

$4\frac{1}{2}N =$ 1*st* income,

or $69N = \frac{92}{6}$ of 1*st* income;

$\therefore 3640 - 69\frac{1}{2}N = 78\frac{3}{4} \times \frac{92}{6}$,

$69\frac{1}{2}N = 3640 - 1207\frac{1}{2} = 2432\frac{1}{2}$;

$\therefore N = 4865 \div 139 = 35$; hence $3640 \div 35 = 104$. *Ans.*

EXERCISES 9.

1. A sells some stock and B buys it, each paying the broker $\frac{1}{8}$ per cent. If the broker makes £8 8*s.* 3*d.* by the transaction, what is the amount of stock dealt with?

2. A gentleman transfers £3000 of 3 per cent. stock at $89\frac{1}{4}$ to the $3\frac{1}{2}$ per cents. at 102. Find the alteration in his income.

3. If a person invests £9065 in the 4 per cents. at $97\frac{1}{8}$, and, on their rising to par, sells out, and invests

the money in Consols at $5\frac{1}{2}$ per cent. discount, what amount of Consols does he obtain?

4. A person possessing £2050 in the 4 per cents., when they are at $112\frac{1}{2}$ transfers three-fifths of that amount to the 3 per cents. at 82. What alteration is made in his income?

5. If I buy 6 per cent. bonds at 120, what interest shall I have on the investment?

6. What sum, if invested in $4\frac{1}{2}$ per cent. stock at $101\frac{1}{4}$, would obtain £16 8*s*. more income than if invested in 4 per cent. stock at par?

7. A man's income from the 3 per cent. Consols is £750. He sells out half his stock at 96, and invests it in other securities at 120. If his income be increased by £50, what interest do these securities pay?

8. A person having a certain sum of money to invest, finds that a railway preference 5 per cent. stock at $117\frac{1}{2}$ will yield him £29 more interest annually than an investment in the 3 per cent. Consols at $92\frac{1}{4}$. How much money has he to invest?

9. A man's net income, derived from money invested in the 3 per cents., amounts, after paying income-tax of 5*d*. in the pound, to £452 7*s*. 6*d*. He sells out the whole at $78\frac{1}{2}$, and with the proceeds buys India 4 per cent. stock at $102\frac{1}{2}$. Find, within a penny, the alteration in his income.

10. A person has a sum to invest which will buy £187 10*s*. more of 3 per cent. than of $3\frac{1}{2}$ per cent. stock, when the market price of the former is 84 and that of the latter 96. What difference of income is there from investing in these securities?

11. What description of stock at $80\frac{3}{4}$ yields $5\frac{5}{19}$ per cent. on investments?

12. A proprietor of $3\frac{1}{2}$ per cent. stock sells out at $92\frac{3}{4}$ and realises £18550, of which he invests two-fifths in the 4 per cents. at 96, and the remainder in the 3 per cents. at 90. Find the alteration in his income.

13. A holder of 3 per cent. stock, on receiving a half-year's dividend, invests it in the same stock at $93\frac{3}{4}$. His next half-year's dividend is £381. What amount of stock had he at first?

14. I invested £2345 in $3\frac{1}{4}$ per cent. stock at $87\frac{1}{2}$, and on the price rising to 90 I sold out and invested the cash proceeds in the 3 per cents., whereby my income was made 14*s.* less than before. At what price did I buy into the 3 per cents.?

15. A person who has £10257 10*s.* 3 per cent. stock, calculates that by selling it and investing the cash in $3\frac{1}{4}$ per cent. stock at $93\frac{1}{4}$, he can increase his annual income by £10 9*s.*, but before he can effect the exchange each stock rises $\frac{1}{4}$ per cent. By how much is his income really increased?

16. The sum of £5170 was invested in the 4 per cents.; and when the market price rose $3\frac{1}{8}$ the owner sold out, and with the proceeds bought $3\frac{1}{4}$ per cent. stock at 77, thereby making his income better by £5 9*s.* $4\frac{1}{2}$*d.* Find the price of the 4 per cents. at first.

CHAPTER IX.

INTERIOR AREAS OF RECTANGULAR ROOMS, ETC.

OF those portions of Mensuration that should be recognised in common Arithmetic, the most essential are circles and rectangular areas and volumes; and accordingly many examination papers in Arithmetic include questions about them. The floor and walls of apartments are especially made subjects of calculation, and we will here exemplify the most usual ways in which Arithmetic deals with such subjects.

Ex. 1. What will be the cost of painting the walls of a room, at 1*s.* 7*d.* per square yard, the length being 19 feet $10\frac{1}{4}$ inches, the breadth 16 feet $1\frac{3}{4}$ inches, and the height 10 feet 3 inches?

It is usual, as in this question, to neglect deductions for ordinary breaches in the continuity of the walls, such as a fireplace or a window.

Do not calculate separately the areas of the sides and ends of the room, but let them be dealt with as if they comprehended one rectangle, of which the breadth is the room's height, and the length is its circuit, that is, twice the sum of its length and width.

Here, then, the circuit of the room is (19 ft. $10\frac{1}{4}$ in. + 10 ft. $1\frac{3}{4}$ in.) $\times 2 = 36$ ft. $\times 2 = 72$ ft., and the height is $10\frac{1}{4}$ ft. Therefore, the area supposed to be painted is $= 72 \times 10\frac{1}{4}$, or 18 sq. ft. $\times$ 41, or 2 sq. yds. $\times$ 41. Accordingly, the cost is $19d. \times 82 = 1558d. = 129s.\ 10d.$ $= £6\ 9s.\ 10d.$ *Ans.*

Ex. 2. How many yards of paper, 1·7 foot wide, will be required to hang the walls of a room that is 23·22 feet long, 16·9 feet wide, and $13\frac{3}{4}$ feet high, neglecting deductions for door, windows, &c.?

Circuit of the room $= (23{\cdot}22 + 16{\cdot}9)$ ft. $\times 2 = 80{\cdot}24$ ft. Now the whole area of the paper must be equal to that of the walls it is to cover; and therefore the area of the walls divided by the width of the paper will give the length of the paper, which will be reduced to yards by dividing by 3; thus—

$$\frac{80{\cdot}24 \text{ sq. ft.} \times 13\frac{3}{4}}{1{\cdot}7 \text{ ft.} \times 3} = \frac{2006 \times 55}{170 \times 3} = \frac{118 \times 11}{6}$$

$= 216\frac{1}{3}$ yds. *Ans.*

Ex. 3. What is the cost of papering the walls of a room that is 25 feet 9 inches long, 19 feet 7 inches wide, and 14 feet high, the paper being $10\frac{5}{8}$ inches wide, and costing $4\frac{1}{2}d.$ a yard?

Circuit $= (25 \text{ ft. } 9 \text{ in.} + 19 \text{ ft. } 7 \text{ in.}) \times 2 = 90\frac{2}{3}$ ft.; then the area of walls or paper being $= 90\frac{2}{3}$ sq. ft. $\times 14$, and the width of the paper $= 10\frac{5}{8}$ in. $= \frac{85}{96}$ of a foot; the length of the paper in feet will be its area in square feet divided by its width in feet; and then, reducing to yards and calculating at $4\frac{1}{2}d.$ a yard, we have

$$\frac{4\frac{1}{2}d. \times 90\frac{2}{3} \times 14 \times 96}{85 \times 3} = \frac{9d. \times 272 \times 7 \times 32}{85 \times 3}$$

$= \frac{1}{5}$ of $(3d. \times 16 \times 224) = 2150\frac{2}{5}d. = £8\ 19s.\ 2\frac{2}{5}d.$ *Ans.*

Ex. 4. In order to paper a room $27\frac{3}{8}$ feet long, $17\frac{1}{4}$ feet wide, and $15\frac{5}{6}$ feet high, 50 pieces of paper, of 12 yards long and $9\frac{1}{2}$ inches wide, were bought, to allow for probable waste. What was the quantity of waste?

Here $(27\frac{3}{8} + 17\frac{1}{4}) \times 2 \times 15\frac{5}{6}$ will give the area of the paper in square feet, and this $\div 9\frac{1}{2}$ inches, or $\frac{12}{24}$ of a

foot, will give the required length of paper in feet, to be reduced then to yards by dividing by 3: thus,

$$\frac{44\frac{5}{8}\times 2\times 15\frac{5}{6}\times 24}{19\times 3} = \frac{357\times 95}{19\times 3} = 119\times 5 = 595 \text{ yards};$$

which, subtracted from 12 yards × 50, gives 5 yards. *Ans.*

Ex. 5. The area of each of the longer walls of a rectangular room is 330 square feet, that of each the other walls is 220 square feet, and that of the floor is 384 square feet. How many yards of paper, 9 inches wide, are required to cover the walls, deducting a twenty-fifth of the whole area for the door, fire-place, &c.?

The length multiplied by the height = 330 sq. ft.;
the breadth „ „ height = 220 „
the length „ „ breadth = 384 „

Hence the number of cubic feet in the volume of the room = breadth × 330 = length × 220 = height × 384;

∴ the breadth = $\frac{220}{330}$ or $\frac{2}{3}$ of the length;

accordingly the product of length and breadth = $\frac{2}{3}$ of the square of the length; hence, $\sqrt{384\div\frac{2}{3}} = \sqrt{576} = 24$ ft. the length of the room; so then $\frac{2}{3}$ of 24 = 16 ft. is the width, and $330\div 24 = 13\frac{3}{4}$ ft. the height.

Now to find the length of paper required for $\frac{24}{25}$ of the area of the walls, we have

$\frac{24}{25}$ of $(24+16)\times 2\times 13\frac{3}{4}\div$ the width of the paper, viz. $\frac{3}{4}$ of a foot, and then, reduced to yards,

$$= \frac{80\times 13\frac{3}{4}\times 24}{25\times\frac{3}{4}\times 3} = \frac{20\times 55\times 96}{25\times 9} = \frac{1}{3} \text{ of } (4\times 11\times 32)$$

$= 469\frac{1}{3}$ yds. *Ans.*

Ex. 6. There are two rectangular wells, one of them 5 feet by 4 feet in section, the other 6 feet by $3\frac{1}{2}$ feet. A hose which fills the first in 4 minutes would take 7

minutes to fill the other; and if the water that would fill the smaller well were poured into the larger one, it would stand 15 feet below the top. What is the depth of each well?

Call the depth in feet of the smaller well D.

The contents of the smaller well $= 5 \times 4 \times D$ cubic feet, that of the larger $= \frac{7}{4}$ of $5 \times 4 \times D = 35D$;

$35D - 20D = 6 \times 3\frac{1}{2} \times 15$, or $15D = 315$;

hence the depth of the smaller well is 21 feet; and that of the larger is $= \frac{35D}{6 \times 3\frac{1}{2}} = \frac{5}{3}$ of 21 ft. $= 35$ ft. *Ans.*

EXERCISES 10.

1. What will be the cost of papering a room 27 feet long, 17 feet wide, and $16\frac{1}{4}$ feet high, with paper $\frac{5}{8}$ of a yard wide, at $4\frac{1}{2}d.$ per yard long, neglecting deductions for door, windows, &c.?

2. A room 21 feet 4 inches long, 14 feet 8 inches wide, and 14 feet high is to be papered. How many yards of paper $2\frac{3}{8}$ feet wide is required, if a deduction of 17 square yards be made for the door, &c.?

3. A room is 23 feet 7 inches long, 15 feet 5 inches wide, and $11\frac{1}{2}$ feet high. What would be the cost of hanging its walls with paper 15 inches wide, at $7\frac{1}{2}d.$ per yard, deducting for the door, &c., $\frac{1}{6}$ of the whole area?

4. The cost of the paper necessary to hang the walls of a room $19\frac{1}{2}$ feet long, $17\frac{1}{4}$ broad, and $12\frac{1}{2}$ high, with paper 14 inches wide, is £4 7*s.* 6*d.*; what is the cost of the paper per yard, not reckoning any deductions for door, fireplace, &c.?

5. The cost of papering a room 30 feet $8\frac{1}{2}$ inches long, 17 feet $3\frac{1}{2}$ inches broad, and $12\frac{1}{2}$ feet high, at $5\frac{1}{2}d.$ per yard, is £7 6*s.* 8*d.* Find the width of the paper.

6. A room is half as long again as it is wide, the circuit of its walls being 30 yards; and the cost of hanging its walls with paper 10 inches wide, at 3*d.* a yard, is £6 10*s.* 6*d.* Find the dimensions of the room.

7. To whiten the ceiling of a room cost £1 17*s.* $9\frac{3}{4}d.$, at the rate of $7\frac{1}{2}d.$ per square yard, and to paint the walls of the same amounted to £29 6*s.* 8*d.*, at 3*s.* 4*d.* per square yard. Find the height of the room, its length being exactly twice its width.

8. Two rectangular rooms of the same height were papered: one of them 16 feet long and 14 feet wide, the other 14 feet long and 12 feet wide. The paper was $1\frac{3}{4}$ feet wide, and cost 2*s.* 9*d.* per piece of 12 yards, and the hanging cost 9*d.* per piece. The whole expense was four guineas. Find the height of the rooms.

9. The length, breadth, and height of a rectangular room are in the proportion of 8 : 6 : 5. Paper 15 inches wide to cover the walls would cost £4 7*s.* 6*d.*, at 4*s.* 6*d.* per piece of 12 yards, not reckoning for any waste. What will be the cost of laying the floor with carpet $2\frac{1}{4}$ feet wide, at 3*s.* 9*d.* a yard?

CHAPTER X.

OF LATITUDE AND LONGITUDE.

(1) *Length of a Degree of Longitude.*

1. THE 360th part of the circumference of any circle is called a *degree*, and therefore varies in absolute length according to the length of the circumference. The Equator being a circumference of about 24876 miles, the length of one degree of it is equal to $24876 \div 360 = 207{\cdot}3 \div 3 = 69\frac{1}{10}$ miles; but a circumference parallel to the Equator, and drawn so as to pass through London, would measure only 15480 miles, and one degree of it is equal to $15480 \div 360 = 129 \div 3 = 43$ miles.

2. Assuming the earth to be a sphere, each meridian circle is regarded as equal in length to the Equator, and therefore all degrees of latitude are 69 miles long. *Latitude*, then, is distance, in equatorial or maximum degrees, north or south from the Equator; but Longitude is distance in degrees of various lengths, east or west from the first meridian. Latitude is measured along a meridian of longitude; longitude along a parallel of latitude.

Thus, London is in the latitude of 51° 30′ N., and Leipzig is in nearly the same latitude; so that 69 miles × $51\frac{1}{2}$, or about 3550 miles, is the distance of

each of these places from the Equator; but although Leipzig is in about $12\frac{1}{2}$ degrees of longitude east from London, these degrees are only 43 miles long, and the distance between the places is 43 miles $\times$ $12\frac{1}{2}$, or about 537 miles.

On the parallel of 60° a degree is half as long as an equatorial degree, as will be seen from the following table of the length of a degree of longitude at every ten degrees from the Equator.

Length of a Degree of Longitude.

In Lat.	0°	it is	69 miles.	In Lat.	50°	it is	44 miles.
„	10°	„	68 „	„	60°	„	$34\frac{1}{2}$ „
„	20°	„	65 „	„	70°	„	$23\frac{1}{2}$ „
„	30°	„	60 „	„	80°	„	12 „
„	40°	„	53 „	„	90°	„	0 „

But we have detected an artificial means by which the contents of the above table may be very easily remembered. Observe that for 80°, 70°, 60°, the lengths are *one two, two three, three four.* (The extra half mile for 70° and 60° is often neglected.) Then, for 50°, 40°, &c., the length may be found *by subtracting the square of the tens' figure in the number of degrees from* 69, as follows:—

On the Parallel of	50°	the length is	$69-5^2=44$ mi.
„	40°	„	$69-4^2=53$ „
„	30°	„	$69-3^2=60$ „
„	20°	„	$69-2^2=65$ „
„	10°	„	$69-1^2=68$ „

3. Degrees of longitude vary in length as the cosine of the latitude. (A demonstration of this is given in a note at the foot of the next page.) Hence 69 miles multiplied by the natural cosine of any latitude, as

found in books of mathematical tables, will give the length in miles of a degree in that latitude.[1]

But the length of a degree for every tenth parallel being known, that for any other parallel can be found with sufficient accuracy by proportional interpolation.

Examples.—Required the length of a degree of longitude for the parallels of (i.) 22° 12′, and (ii.) 46° 27′.

(i.) 22° 12′ is between 20° each 65 mi., and 30° each 60 mi.

The 10° diff. of latitude makes a diff. of 5 miles.

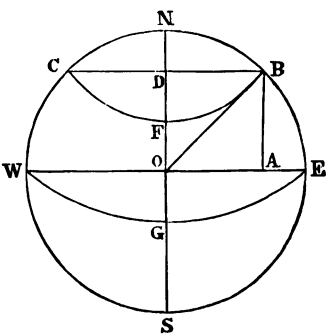
Hence, $\frac{30 \text{ deg.} - 22\frac{1}{2} \text{ deg.}}{10 \text{ deg.}}$ of 5 miles = 3·9 miles to be added to the 60 miles, making about 64 miles. *Ans.*

(ii.) 46° 27′ is between 40° each 53 mi., and 50° each 44 mi.

The 10° diff. of latitude makes a diff. of 9 miles.

Hence, $\frac{50 \text{ deg.} - 46\frac{9}{20} \text{ deg.}}{10 \text{ deg.}}$ of 9 miles = 9 miles × ·355 = 3·2 miles, to be added to the 44 miles, making about 47 miles. *Ans.*[2]

[1] Let WNES represent the circumference of the earth, N and S the poles, WGE the equator, and WE its diameter, CB the diameter of CFB, a parallel at the distance EB, say 45°.



The circumferences of circles are to one another as the radii; hence, the length of a degree on WGE is to that of a degree on CFB as OE is to DB, or as OB is to OA, that is, as 1 is to cosine of the angle BOE; and the cosine of 45° being $\frac{1}{2}\sqrt{2}$ = ·7071, we have 69 miles × ·7071 = 48¾ miles, *nearly*, for the length of a degree of longitude in the latitude of 45°.

[2] *Verifications*: (i.) 69 × cos. 22° 12′ = 69 × ·926 = 63·894.
(ii.) 69 × cos. 46° 27′ = 69 × ·6889 = 47·5.

4. The length in miles of a degree of longitude being given, the latitude may be similarly found.

Examples.—The lengths of a degree of longitude for two different latitudes are 63·9 and 47·2 miles. Find each latitude.

(i.) 63·9 miles is between 60 and 65 miles.

A degree is 60 miles in latitude 30°, and 65 miles in latitude 20°.

A difference of 5 miles corresponds to a difference of 10° of latitude.

Hence, $\frac{65 \text{ mi.} - 63{\cdot}9 \text{ mi.}}{5 \text{ mi.}}$ of $10° = 2\frac{1}{5}$ deg. to be added to the 20°, making 22° 12′. *Ans.*

(ii.) 47·2 miles is between 44 and 53 miles.

A degree is 44 miles in latitude 50°, and 53 miles in latitude 40°.

A difference of 9 miles corresponds to a difference of 10° of latitude.

Hence, $\frac{53 \text{ mi.} - 47{\cdot}2 \text{ mi.}}{9 \text{ m.}}$ of $10° = \frac{58}{90}$ of $10° = 6\frac{4}{9}$ deg., or 6° 27′ to be added to the 40°, making 46° 27′. *Ans.*

5. The rapidity with which a point on the earth's surface is carried round by the earth's rotatory motion depends on latitude. It is less and less, the farther north or south the point is from the Equator. People on the Equator itself are carried round 69 miles × 360 in 24 hours, which is at the rate of 69 × 15, or 1035, miles an hour. To find, then, the velocity per hour for any parallel, we have only to multiply the length of a degree of the parallel by 15. Thus, the inhabitants of London, in lat. 51° 30′, where a degree of longitude is 43 miles, are whirled round 43 × 15 = 645 miles an hour.

EXERCISES 11.

Find the length of a degree of longitude at each of the following places, the latitude of each being as annexed.

1. Delhi . .	28° 35′ N.	6. Bahia .	12° 45′ S.
2. Valparaiso	23° 8′ S.	7. Rome .	41° 52′ N.
3. Iceland .	65° 0′ N.	8. Paris . .	48° 50′ N.
4. Cape Horn	56° 0′ S.	9. Spitzbergen	78° 0′ N.
5. Oporto .	41° 0′ N.	10. Bombay .	19° 8′ N.

Find the latitudes of the following places, the length of a degree of longitude at each being as annexed. Find also the rate per hour at which the inhabitants of each are carried by the diurnal revolution.

11. Rio Janeiro .	63½ mi.	14. Alexandria .	59 mi.
12. Monte Video .	56¾ „	15. Melbourne .	54½ „
13. Vienna . . .	46 „	16. Thurso . . .	35·8 „

17. At Surat, in Hindostan, the inhabitants are carried by the earth's rotation half as fast again as at London, of which the latitude is 51° 30′. Find the latitude of Surat.

18. In what latitudes are the inhabitants carried round one-third as fast again as at Manitoba, where the latitude is 49° 40′ N.?

(2) *Distance between Places on the Earth's Surface.*

6. When two places are on the same meridian, the distance between them is, of course, equal to the product of 69 miles by the number of degrees in the difference of their latitudes. When both are on the same parallel, the distance between them is equal to

the length in miles of a degree of that parallel multiplied by the number of degrees in the difference of their longitudes.

Ex. 1. The longitude of Quebec is 71° 18′ W., that of Vienna is 16° 24′ E., and both places are very nearly in the latitude of 48° N. Find the least distance between them.

In lat. 40° a degree = 53 miles, in lat. 50° it is 44 miles.

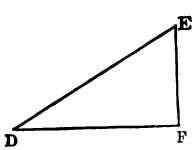
$\frac{50°-48°}{10°}$ of (53−44) mi. = 1·8 mi. to be added to the 44 miles.

45·8 mi. × (71° 18′ + 16° 24′) = 45·8 mi. × 87·7 = 4016 miles. *Ans.*

7. But when the degrees of latitude for two places are different quantities, and their degrees of longitude are also different, a tolerably near approximation to the *direct* distance between them may be found by treating it as the hypotenuse of a right-angled triangle.

Ex. 2. Find, approximately, the shortest distance of Petersburg from Paris, the former being in lat. 60° N., long. 30° E., and the latter in lat. 49° N., long. 2° E.

Let D be the position of Paris, E that of Petersburg; D F the difference of their longitudes = the length in miles of 28° on the parallel of 49°.

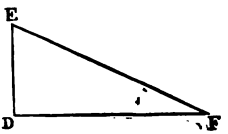


$\frac{50°-49°}{10°}$ of (53−44) mi. = ·9 of a mile to be added.

D F is therefore = 44·9 mi. × 28, which reduced to maximum degrees = 1257 ÷ 69 = 18·2 deg.; then as F E the difference of the latitudes = 11 such degrees, we have D E = $\sqrt{(18{\cdot}2^2+11^2)}$ = $\sqrt{452}$ = 21·2 deg. of 69 mi. = 1460 miles. *Ans.*

Ex. 3. Find, approximately, the least distance between London, lat. 51° 30′ N., long. 0°, and Bombay, lat. 20° N., long. 73° E.

Let E be the position of London, F that of Bombay; D F the difference of their longitudes = length in miles of 73° in lat. 20° = 65 mi. × 73°; and this reduced to 69 mile degrees = 68·77 deg.



Accordingly, we have D F = 68·77 deg., and D E = $51\frac{1}{2} - 20 = 31\frac{1}{2}$ deg.;
hence E F = $\sqrt{(68{\cdot}77^2 + 31{\cdot}5^2)} = \sqrt{5721} = 75{\cdot}64°$ of 69 miles = 5220 miles. *Ans.*

*** The *travelling* distance run in a ship's course would be upwards of 6000 miles.

EXERCISES 12.

1. The latitude of Stettin is 53° 25′ N., that of Malta is 35° 52′ N., and both places are very nearly on the meridian of $14\frac{1}{2}$ degrees E. Find the least distance between them in miles.

2. The longitude of Gibraltar is 5° W. and that of Malta is 14° 30′ E., and both places are very nearly on the parallel of 36° N. Find their shortest distance from each other in miles.

3. The latitude of Paris is 49° N., that of Barcelona is 41° N., and both places are in long. 2° E. How many miles are they apart?

4. Find approximately, in miles, the direct distance of Rome from Naples, the former being in lat. 41° 52′ N., long. 12° 30′ E., the latter in lat. 40° 50′ N., long. 14° 14′ E.

5. Lisbon is in lat. 38° 42′ N., long. 9° 10′ W. Find in miles its least distance across to Barcelona, lat. 41° N. long. 2° E.

6. How many miles is the shortest distance between Cape Town, in lat. 34° S., long. 18° 30′ E., and Cape Horn, in lat. 56° S., long. 67° W.

7. Find the direct distance between the Island of Anticosti and Nootka Sound, in Vancouver Island, the former being in longitude 63° W. and the latter in longitude 126° 35′ W., and both being in latitude 49° 30′ N.

8. Find the least distance in miles between the Lizard, in lat. 49° 57′ N., long. 5° 14′ W., and Rio Janeiro, in lat. 23° S., long. 42° 30′ W.

9. What is the breadth of France between Bordeaux, in long. 0° 34′ W., and Mont Cenis, in long. 6° 55′ E., both being in lat. 45° N.?

10. Calculate the length of the Red Sea, viz. from Suez, in lat. 30° N., long. 32° 35′ E., to Bab-el-mandeb Straits, in lat. 12° 50′ N., long. 43° 10′ E.

11. Calculate the direct distance in miles between Mocha, in lat. 13° 30′ N., long. 43° 20′ E., and Buenos Ayres, in lat. 34° 37′ S., long. 58° 30′ W.

12. How many miles distant is Rome from Paris, Rome being in lat. 41° 52′ N., long. 12° 30′ E., and Paris in lat. 48° 50′ N., long. 2° 20′ E?

13. New York is in lat. 41° N., long. 73° 55′ W., and Philadelphia is in lat. 40° N., long. 75° 15′ W. How many miles from one to the other?

14. How far is New York from Liverpool, the former being in lat. 41° N., long. 74° W., and the latter in lat. 53° N., long. 3° W.?

(3) *Difference in Time of Day caused by Difference of Longitude.*

8. The sun in his apparent motion round the earth, *from east to west*, completes a revolution in 24 hours, and therefore advances $360° \div 24 = 15°$ in an hour, which is 1 degree in 4 minutes of time. Hence, a place that is 1 degree of longitude *farther to westward* than another will be 4 minutes *earlier* in its *time of day* than the other, and a place 2 degrees of longitude *farther to eastward* than another will have its *time of day* 8 minutes *later* than the other. Thus, when it is noon at London, it is 48 minutes past 11 A.M. at Liverpool, that is, 12 minutes earlier, because Liverpool is 3° longitude *farther west* than London, and 4 minutes $\times 3 = 12$ minutes.

Accordingly, the rule for finding the difference of time between two places may be stated as follows:—

Note the difference of their longitudes. This difference is found by subtraction, when both places are in East or both in West longitude; but by addition, when one of the places is in East, and the other in West longitude.

And now, multiplying 4 minutes by the degrees in the difference will give the *minutes earlier* for a place farther west, *later* for a place farther east.

Ex. 1. When it is a quarter to 4 P.M. at Malta, long. 14° 25′ E., what is the time of day at Cape Town, long. 18° 27′ E.?

Cape Town is *farther east* by $4\frac{1}{30}$ degrees; hence 4 minutes $\times 4\frac{1}{30} = 16$ minutes *later*; therefore the time required is 3 hrs. 45 min. + 16 min. = 1 minute past 4 o'clock P.M. *Ans.*

Ex. 2. What is the time at Mexico, long. 99° 5′ W., when it is 11^h 10′ A.M. at Madras, long. 80° 15′ E.?

Mexico is farther west by $179\frac{1}{3}$ degrees;
$\therefore$ 4 minutes $\times 179\frac{1}{3} = 717$ minutes $= 11$ hrs. 57 min. earlier $= 47$ minutes before the preceding midnight $= 11$ hrs. 13 min. P.M. *Ans.*

Ex. 3. What is the time at Teheran, long. 51° 4′ E., when it is 4^h 45′ A.M. at Rome, long. 12° 30′ E.?

Teheran is 38° 34′ farther east; hence 4 min. $\times 38\frac{17}{30}$ $= 154$ minutes $= 2$ hrs. 34 min. later $= 19$ minutes past 7 A.M. *Ans.*

*** The preceding examples show how easily we may ascertain the longitude of any place of which the time is given, when the longitude and time at another place are given.

Ex. 4. When it is 20 minutes past 11 P.M. at Rio, long. 42° 30′ W., it is $47\frac{1}{3}$ minutes past 1 A.M., viz. 2 hrs. $27\frac{1}{3}$ min. later, at St. Helena. Find the longitude of St. Helena.

2 hrs. $27\frac{1}{3}$ min. $= 147\frac{1}{3}$ min., which, at 4 min. for a degree, $= 147\frac{1}{3} \div 4 = 36°\ 50′$ farther east; which makes the required longitude $= 42°\ 30′ - 36°\ 50′ = 5°\ 40′$ W. *Ans.*

Ex. 5. A person at sea finds that it is noon on shipboard when it is 3 P.M. at London. What is the ship's longitude?

3 hours later $= 180$ min., which at 4 min. for a degree $= 180 \div 4 = 45°$ W. *Ans.*

EXERCISES 13.

1. When it is noon at London, (i.) what is the time at New York, long. 73° 55′ W., (ii.) what at Petersburg, long. 30° 10′ E., (iii.) and what at Constantinople, long. 29° E.?

2. When it is 2 o'clock P.M. at Calcutta, long. 88° 30′ E., what is the time at Coventry, long. 1° 30′ W.?

3. When it is noon at Barcelona, long. 2° 30′ E., what is the time at Sydney, long. 151° 11′ E.?

4. When it is 8 o'clock P.M. at the Mauritius, long. 57° E., what is the time at Lima, long. 76° 56′ W.?

5. What is the time at Shanghai, long. 120° E., when it is 6 o'clock A.M. at Yokohama, long. 139° 45′ E.?

6. When it is 5 P.M. at Cairo, long. 31° 25′ E., what is the time of day at Lisbon, long. 9° 10′ W.?

7. What o'clock is it at New Orleans, long. 90° W., when it is 3 o'clock P.M. at Amsterdam, long. 4° 52′ E.?

8. A person at sea observes that the sun is over his meridian when, by a chronometer indicating London time, it is only half-past 10 A.M. What is his longitude?

9. What places have noon, when at Cape Town, long. 18° 27′ E., it is 15 minutes after noon?

10. What is the longitude of Petersburg, the time there being half-past 1 o'clock when it is half-past 11 o'clock at London?

CHAPTER XI.

MISCELLANEOUS QUESTIONS.

REFERRING to what has been said in the Introduction (foot of page 3) respecting the comparative merits of Algebra and Arithmetic, we shall here exemplify that statement by giving algebraic and arithmetical solutions of the following problem :—

A and B had each a certain sum of money, from which they have deposited stakes amounting to 100 *guineas, to be competed for in a rowing match between them. A's stake is to B's in the proportion which B's original money bore to A's original money. Now, if A win, A's money will be made equal to* 5 *times the half of what B will have remaining; but if B win, B's money will be made equal to* 15 *times the* 13*th part of what A will have remaining. What sum had each at first?*

Algebraic Solution.

Let x and y denote, respectively, the numbers of pounds A and B had at first; then, according to the inverse proportion, vy may denote A's stake, and vx B's stake.

If A win, A will have $x + vx$, and B $y - vx$;
if B win, A will have $x - vy$, and B $y + vy$.
Hence we have the equations :—

$2x + 2vx = 5y - 5vx$, or $2x - 5y = -7vx$,

$15x - 15vy = 13y + 13vy$, or $15x - 13y = 28vy$;

From the second of these subtract four times the first: $7x + 7y = 28v(x + y)$; $\therefore 28v = 7$, and $v = \frac{1}{4}$; whereby the first equation becomes $2x - 5y = -1\frac{3}{4}x$; hence $y = \frac{3}{4}x$; but $vx + vy = 105$, or $x + y = 420$, that is, $1\frac{3}{4}x = 420$; whence A's original money £240, B's £180. *Ans.*

Arithmetical Solution.

In attempting an arithmetical solution, we engage in a more difficult, but at the same time a more intellectual exercise. We should first note the factorial relations that are given. We have 5 : 2, the ratio of A's original money *plus* B's stake to B's original money *minus* B's stake. This is a division of the *joint original money* into two parts. We have also 15 : 13, the ratio of B's original money *plus* A's stake to A's original money *minus* A's stake. This is another division of the *joint original money* into two parts. The joint original money, therefore, being referred to as unity,—

$\frac{5}{7}$ represents A's original money + B's stake.
$\frac{15}{28}$ „ B's „ + A's „

the sum $\frac{5}{4}$ denotes £105 more than the joint original money;

$\therefore$ $\frac{1}{4}$ of that joint money is £105, and, accordingly, A and B had together £420.

It now appears that B's stake was equal to $\frac{1}{4}$ of A's original sum, and hence

$\frac{5}{7}$ represents A's original money + $\frac{1}{4}$ of ditto.

$\therefore$ A's original money = $\frac{5}{7} \div 1\frac{1}{4}$, or $\frac{4}{7}$ of the £420 = £240, and B's = £420 − £240 = £180. *Ans.*

*** Here Arithmetic presents a better solution than Algebra. It is true that in solving the question algebraically, we might employ more artful reasoning

than is contained in the algebraic solution given above, but we could do so only through *cultivated arithmetical skill* beyond what the question, as an algebraic one, *necessarily requires*. Thus we might have written:—

If A win, he will have $x + v\,x = \frac{5}{7}(x + y)$;
if B win, he will have $y + v\,y = \frac{15}{28}(x + y)$;
comparing these equations gives $x : y :: \frac{5}{7} : \frac{15}{28}$; and adding them gives $v + 1 = \frac{5}{7} + \frac{15}{28}$.

EXERCISES 14.

1. Divide 60 into two such parts, that one of them increased by 7 shall be equal to the other diminished by 10.

2. A man having a basket of walnuts is selling them at 8 a penny; and I could buy all his store at that rate, if the money I have were 2*d.* more. He agrees, however, to let me have them 10 a penny, for which my money is 4*d.* more than sufficient. How many walnuts do I buy?

3. A invests 2*s.* 4*d.* and gains 6*d.*; B invests 3*s.* 8*d.* and gains $10\frac{1}{2}d.$ By how much per cent. does B's gain exceed A's?

4. Find the time of day at Barbadoes, long. 59° 40′ W., when it is 8 o'clock P.M. at London.

5. A can do a piece of work in 16 days, B could do it in 15 days, C in 10 days. In what time could the three working together accomplish it?

6. Find the difference between the mercantile and the true discount on a bill of £340, payable six months hence, at 4 per cent. per annum.

7. If 12 men or 17 women can finish a piece of

work in $9\frac{1}{2}$ hours, in what time could 9 men and 11 women together finish it?

8. A woman having bought a number of oranges at the rate of 3 for 2*d.*, and as many more at the rate of 4 for 3*d.*, sold the whole at the rate of 7 for 5*d.*, and gained 4*d.* How many dozen did she buy?

9. A has 47 florins and a sovereign, B has 47 sovereigns and a florin. What sum must A give to B that the latter may have ten times as much money as the former?

10. If to perform a certain piece of work in an hour would require 9 labourers each as expert as A, or 11 each as expert as B, in what time could A and B together do it?

11. A lady, examining her purse to see how much she should give to each of a number of poor women, finds that she has 15*d.* too little to give 3*s.* 9*d.* to each, but 18*d.* more than sufficient to give each 3*s.* 6*d.* How many poor are there?

12. A and B have together £87, and C has £14 more than A, and £30 less than B. What sum has each?

13. Divide the number 100 into two such parts that 7 times one shall be 10 less than the other.

14. If 9 pears be worth as much as 20 apples, and a dozen of each be together worth 1*s.* $2\frac{1}{2}$*d.*, how many pears can be had for 10*d.*?

15. When it is 53 minutes past 1 o'clock A.M. at Berlin, long. 13° 25′ E., what is the time of day at London?

16. A congregation of 1000 persons consisted of men, women, and children; two-fifths of the whole were women, and there were thrice as many men as children. Find the number of each class.

17. A can perform a piece of work in $5\frac{1}{4}$ days, B could do it in $6\frac{1}{8}$ days, and A, B, and C together in $2\frac{1}{10}$ days; in what time could C alone do it?

18. What fraction of any sum of money is the difference between the interest and the discount on that sum, for 7 months, at 5 per cent. per annum?

19. At what time between 6 and 7 o'clock do the hour and minute hands of a watch form an angle of 60°?

20. If 9 men, or 14 women, or 21 boys, could perform a piece of work in $7\frac{1}{4}$ hours, in what time could 1 man, 1 woman, and 1 boy together perform it?

21. M has £56 4*s.* and N has £54 6*s.*; and N, by now paying a debt he owes to M, has remaining a sum equal to two-thirds of what M has. What was N's debt?

22. When it is 4 o'clock A.M. at Rome, long. 12° 30′ E., what o'clock is it at Ispahan, long. 51° 58′ E.?

23. A company of men, women, and boys have the sum of £9 19*s.* distributed among them, each man receiving 5*s.* 6*d.*, each woman 3*s.* 6*d.*, each boy 1*s.* 6*d.* The women are in number half as many again as the men, and a third as many again as the boys. How many persons are there of each class?

24. Calculate the length of a degree of longitude in miles at Behring's Strait, on the Arctic Circle, lat. $66\frac{1}{2}$ degrees N.

25. A man leaves $\frac{4}{9}$ of his property to his son, one-fourth of the remainder to his sister, and the rest to his widow. The son's share being £150 more than the widow's, what was the sister's share?

26. Two persons starting from the same point, and walking in opposite directions, are $25\frac{1}{2}$ miles apart in 3 hours; but if they had walked in the same direction

they would have been 6 miles apart in 4 hours. Find their rates per hour.

27. Three persons, A, B, C, jointly own an estate, their shares being as 4, $2\frac{1}{2}$, and $1\frac{1}{2}$. A sells half his share to C, and C sells 100 acres to B, and the shares of B and C are thus made equal. How many acres was each entitled to at first?

28. Divide the number 40 into two parts, so that 9 times the less shall exceed 5 times the greater by 38.

29. How many miles does a degree measure in the latitude of 58° 24′?

30. Twenty minutes after M had set out on a journey on horseback at the rate of 8 miles an hour, N set out from the same place, and, following him on a bicycle at uniform speed, overtook him in 2 hours 20 minutes. At what rate per hour did N ride?

31. At what time between 7 and 8 o'clock will the hour and minute hands of a clock intercept an arc equal to 12 of the minute spaces?

32. Divide 316 yards into two such parts that 23 times the less added to 31 times the greater shall make up 5 miles.

33. A man devised $\frac{5}{12}$ of his estate to one of his sons, and half of the remainder to another, and the surplus to his widow. The difference of the sons' legacies was £784. What did the widow receive?

34. A can do a piece of work in 27 days, B can do it in $29\frac{1}{4}$ days, C in $30\frac{1}{3}$ days, D in $31\frac{1}{2}$ days. In what time could it be done by all four working together?

35. Y and Z enter into partnership, Y with a capital of £640, and Z with a capital of £500; at the end of 5 months Y withdraws a fourth of his capital, but at the end of 6 months Z doubles his invest-

ment. What should be each man's share of a profit of £648 6*s.* 8*d.* at the end of the year?

36. What sum at compound interest will amount to £650 at the end of the first year, and to £676 at the end of the second?

37. What is the area of a right-angled triangle that has the base 28 inches, and the sum of the hypotenuse and perpendicular 98 inches?

38. A father's age at present is equal to 12 times the 5th of his son's, and to 8 times the 3rd of his daughter's; and if the three persons live $11\frac{1}{4}$ years longer, the father's age will then be equal to the sum of the others' ages. Find the present age of each.

39. A can do a piece of work in 18 days, B in 20 days, C in $22\frac{1}{2}$ days. The work is done by the three together, with the exception of B not joining the others till after the first 2 days. In what time is the work done?

40. Water is poured into a cylindrical reservoir, 20 feet in diameter, at the rate of 400 gallons a minute. Assuming 4 gallons of water to measure 1109 cubic inches, find the rate at which the surface of the water rises in the reservoir. The area of a circle $= \frac{11}{14}$ of the square on its diameter.

41. If a train 90 yards long overtake a person walking at the rate of $3\frac{3}{4}$ miles an hour along the line, and pass him completely in 10 seconds, what is the rate of the train per hour?

42. When it is half-past 9 o'clock A.M. at New York, long. 73° 55′ W., in what longitude is it 2 P.M.?

43. What must a merchant gain per cent. on his outlay, in order that $10\frac{5}{7}$ per cent. of his returns may be profit?

44. A, B, C, and D undertake to do a piece of

work. A's rate of working is to B's as 3 : 4, B's is to C's as 5 : 4, C's to D's as 5 : 6. A could do it alone in 60 hours. Find in what time each of the other three would do it alone, and in what time all four working together would do it.

45. A woman sold 90 apples, some at the rate of 3 for $2\frac{1}{2}d.$, and the rest at 8 for $6\frac{1}{2}d.$, and she found that those sold at the latter price brought her just $4d.$ more than the others did. How many were sold at each rate?

46. The sum of £7 11*s.* 4*d.* is divided among some men, women, and boys, there being 6 more women than men, and 4 more boys than women. The men get each 5*s.* 8*d.*, the women 3*s.* 10*d.*, and the boys 1*s.* 8*d.* How many are there of each class?

47. A, B, and C enter into partnership with capitals as 1 : 2 : 3. In 6 months A withdraws half of his capital, but he restores it after 3 months more. At the end of 4 months from commencement, B withdraws one-third of his capital; and 2 months later C adds twice as much as B withdrew. How should a profit of £658 10*s.* be divided at the end of the year?

48. A can do a piece of work in 10 days, B could do it in 9 days, C in 12. They all begin it together, but only C continues till the work is finished; A leaving it $3\frac{3}{5}$ days, and B $2\frac{2}{5}$ days, before its completion. In what time is the work done?

49. Water is flowing at the rate of 10 miles an hour through a pipe 14 inches in diameter into a rectangular reservoir 187 yards long and 84 yards wide. Calculate the time in which the surface will be raised one inch. [Reckon the area of a circle equal to $\frac{11}{14}$ of the square on its diameter.]

50. A party of men were employed to do a piece of work, which it was necessary should be finished in 25 days. When, however, the men had wrought 20 days, they had done only $\frac{5}{7}$ of the work, and 3 more men had then to be added to complete the work in due time. How many men were employed at first?

51. A grocer sells tea at 3*s.* 3*d.* per lb. and coffee at 1*s.* 5*d.* Having one day sold $5\frac{1}{2}$ lbs. more of coffee than of tea, he found the amounts of the sales equal. What were the quantities sold that day?

52. How many miles is the shortest distance between Gibraltar, lat. 36° 7′ N., long. 5° 20′ W., and Venice, lat. 45° 20′ N., long. 12° 20′ E.?

53. I have bought 63 dozen of sherry for £78 15*s.*, and intend to reserve part for private use and to sell the remainder at 26*s.* 9*d.* a dozen. What quantity must I sell that my sole pecuniary advantage may consist in having my own reserved quantity at 17*s.* $6\frac{3}{4}$*d.* a dozen?

54. A train leaves London for Brighton at 8 A.M., and meets another which left Brighton for London 35 minutes later, at 9 h. 30 m. A.M., 30 miles from London. Compare the rates of the two trains. The distance from London to Brighton is 50 miles.

55. The difference between the simple and the compound interest on a certain sum of money for 3 years at 5 per cent. per annum is £13 6*s.* $10\frac{1}{2}$*d.* What is that sum of money?

56. A can perform a piece of work in 32 hours, B could perform it in 28 hours, C in 24, D in 21. The work is executed by the four together, with the exception of A and C being each an hour absent during the progress of the work. In what time was the work done?

57. A, B, and C trade with a joint stock of £1534. A's stock is £456, and his time of investment 7 months; B's stock is £546, and his gain £132 12*s.*; C's gain is £108 16*s.*, and his time 8 months. Required A's gain and B's time.

58. A number of reapers, men and women, are paid 2 guineas a day, each man receiving 2*s.* 3*d.* a day, and each woman 1*s.* 6*d.* Had there been 2 men and 11 women more, the two parties would have received equal amounts. How many men and how many women were there?

59. Two trains on the same railway are running past each other, in opposite directions, their rates being 40 and 30 miles an hour. Each has an engine and tender; the first has 12 carriages and the second 17. If we suppose the length of an engine and tender to be 40 feet, the length of a carriage to be 32 feet, and the coupling spaces to be each 5 feet, how much time will elapse, from the moment that the engines meet, till the last carriages of the trains have passed each other?

60. To complete a certain piece of work, A by himself would take 7 fifths of the joint time of B and C, and C thrice as long as the joint time of A and B; moreover, A, B, and C by their united efforts could do the whole in 5 days. In what time could each by himself accomplish it?

61. A son's age is equal to $\frac{1}{12}$ of his father's, but 24 years ago it was equal to $\frac{1}{4}$ of the father's; how many years hence, if both live, will the son's be equal to $\frac{5}{8}$ of the father's age?

62. By selling out £3650 of $3\frac{1}{2}$ per cent. stock at 87, and investing the proceeds in $4\frac{1}{2}$ per cent. stock, a

person finds his income made £28 17*s*. better. At what price does he purchase the $4\frac{1}{2}$ per cent. stock?

63. The sum of £232 10*s*. was divided into two equal numbers of guineas and half-sovereigns, and the guineas being improved at 4 per cent. per annum, simple interest, and the half-sovereigns at $4\frac{2}{3}$ per cent., both for the same period, the whole amount of principal and interest was £239 17*s*. Find the time.

64. I gave £3 10*s*. for two dozen of wine, at different rates per dozen; and by selling one kind at an advance of 25 per cent., and the other at a reduction of 15 per cent., I obtained a uniform price for both. What did each dozen cost me?

65. A and B are partners in trade. A invests $\frac{8}{17}$ of the joint capital for $10\frac{1}{2}$ months, and B receives $\frac{6}{13}$ of the gain. Required B's time.

66. Divide the number 536 into four such parts that adding 12 to the first shall produce the same result as subtracting 19 from the second, multiplying the third by 12, or dividing the fourth by 19.

67. Divide £836 10*s*. into three such sums that 5 months' simple interest on the first at $3\frac{1}{2}$ per cent. per annum shall be equal to 7 months' interest on the second at 4 per cent., and to 8 months' interest on the third at $3\frac{1}{4}$ per cent.

68. A sold a stock of wine to B, who then disposed of it to C, and C sold it to D for £1984 17*s*. A gained 5 per cent., B 6 per cent., C 7 per cent. What did the wine cost A?

69. A mixture is composed of 83 pints of rum and 9 of water; how much rum must be added that 25 pints of the mixture may contain 2 pints of water?

70. What sum of money is such that if $\frac{11}{13}$ of it be

distributed among four persons, and the remainder improved at 5 per cent., simple interest, for 12 years, and then $\frac{23}{40}$ of the amount be distributed as before, there will be £102 left?

71. A railway train, moving with uniform speed, is met by an engine and tender 25 feet long, running 27 miles an hour. The engine and tender pass the train in 4 seconds after meeting it, and, returning shortly afterwards at the same rate, repass the train in 20 seconds after overtaking it. Required the length of the train.

72. I bought 16 cows and 6 score sheep for £465, and then sold the whole for £496 10*s*., at a profit of $7\frac{1}{2}$ per cent. on the cows and 6 per cent. on the sheep. What did they cost me a head?

73. Find three sums of money, the greatest of them exceeding the smallest by £47 10*s*., so that the simple interest on the greatest at 6 per cent. per annum for 2 years shall be equal to that on the smallest at 4 per cent. for 4 years, and equal to that on the third sum at 5 per cent. for 3 years.

74. A circular race-course is 22 yards wide and occupies 12 acres. Find the diameter of the inner circle, reckoning the area of a circle equal to $\frac{11}{14}$ of the square on its diameter.

75. Divide the number 1880 into four such parts that the first increased by 55, the second diminished by 44, the third multiplied by 33, and the fourth divided by 22, shall all make the same result.

76. Divide £1660 into two sums, so that the true discount on one for 4 years at $3\frac{1}{2}$ per cent. per annum may equal the true discount on the other for 6 years at $2\frac{1}{2}$ per cent.

77. The true discount on £324 4*s*. 10*d*. for a certain time is £20 1*s*. 6*d*.; what is the discount on it for a third of the time?

78. A fishmonger lays out £13 19*s*. 2*d*. in the purchase of salmon at 18*d*. a lb. and cod at 5*d*.; and he sells the whole for £16 1*s*. 5½*d*., gaining 20 per cent. on the salmon, but losing 5 per cent. on the cod. How many lbs. of each did he buy?

79. If the manufacturer makes a profit of 20 per cent., the wholesale dealer a profit of 25 per cent., and the shopkeeper a profit of 30 per cent., what was the cost of manufacturing an article that was sold at the shop for 17*s*. 4*d*.?

80. A certain piece of work is done by four men, A, B, C, D. The whole could be done by A alone in 21 hours 22½ minutes, or by B alone in 22 hours 10 minutes, by C in 23 hours 45 minutes, by D in 26 hours 15 minutes. All four work together, with the exception of C being called away for 2 hours. In what time is the work done?

81. A merchant offers to sell me a quantity of goods at 25*s*. 10*d*. a ton with a certain term of credit, or at 26*s*. 3*d*. with 2 months' longer credit. For what ready money did he buy the goods 2 months ago, if he is now demanding a profit of 12½ per cent. on his full outlay, and reckoning the use of money to be worth 10 per cent. per annum?

82. A person invested £5100, part in the 3 per cents. at 85½, and the remainder in the 3½ per cents. at 98; and his joint income from the two investments was £180. What were the sums invested?

83. Divide £5235 into three sums, so that the amount of the first by simple interest in 2½ years at

4 per cent. per annum may be equal to that of the second in $3\frac{1}{3}$ years at 5 per cent., and to that of the third in $4\frac{1}{4}$ years at 3 per cent.

84. The areas of the several faces of a rectangular solid are 57, 27, and 19 square feet. Find its dimensions.

85. A circular lamina, $10\frac{1}{2}$ inches in diameter, has a concentric circle cut out of it, leaving a flat ring of 55 square inches. Find the breadth of the ring. [Area of a circle $= \frac{11}{14}$ of the square on its diameter.]

86. A railway train, having performed a journey at $\frac{5}{7}$ of the proper speed, arrived at the termination 36 minutes behind time. In what time should the journey have been performed?

87. From a full cask containing 96 gallons of spirits a certain quantity was drawn, and the cask filled up with water; then the same quantity of the mixture was drawn, and the cask again filled up with water; and when this was done for the fourth time there remained only $30\frac{3}{8}$ gallons of spirits in the cask. What quantity was drawn off each time?

88. In a large institution, where there is always the same number of inmates to be fed, the contract price of meat rises 20 per cent., and the daily allowance of each person is at the same time reduced from 9 ounces to 8 ounces. If the yearly charge for meat is now £597, what was it before the changes were made?

89. A sells to B 245 yards of linen at 3*s.* $9\frac{3}{4}$*d.* a yard, and at the same time B sells to A $91\frac{1}{8}$ gallons of spirits at 10*s.* $3\frac{1}{2}$*d.*; and to balance accounts B allows A 3 months' longer credit than he asks from A. Required the respective terms of credit, reckoning interest at 5 per cent.

90. The largest of three sums of money exceeds the least by £108, and they are such that the amount of the largest by simple interest for 3 years at $3\frac{1}{2}$ per cent. per annum would be equal to that of the least for 5 years at 6 per cent., and also equal to that of the third sum for 4 years at $4\frac{1}{4}$ per cent. Find the several sums.

91. If 36 men can do $52\frac{1}{2}$ units of work in 4 days, working 7 hours a day, what uniform length of day will 40 men require, so that 69 such units of work may be done in 5 days, supposing that any 5 of the latter party can do in 7 hours as much as any 8 of the former party can do in $5\frac{1}{4}$ hours, and that from the latter party 13 men are to be withdrawn after 2 days' work?

92. There are two vessels, S and T, each containing a mixture of water and wine, S in the proportion of 2 : 3, and T in the proportion of 3 : 7. What quantity must be taken from each, in order to form a third mixture, that shall consist of 5 gallons of water and 11 of wine?

93. A invests a certain sum in $3\frac{1}{2}$ per cent. stock at $82\frac{1}{8}$, and B a sum greater than A's by £70 in $3\frac{1}{4}$ per cent. stock at 81. What sum did A invest, to make his income just 16*s.* 8*d.* more than B's?

94. What vulgar fraction has 96 for the sum of its terms, and ·92 a very near approximation to the decimal value of its square?

95. The cost of laying a room floor with carpet $\frac{3}{4}$ of a yard wide, at 3*s.* 6*d.* the yard, is £11 13*s.* 4*d.* What will be the cost of papering the walls with paper 10 inches wide, at 6*d.* a yard, the side walls being each $37\frac{1}{2}$ square yards, and the end walls each 27 square yards, neglecting deductions for door, windows, &c.?

96. The true discount on £142 12*s.* for a certain time is £3 2*s.* What would be the discount on the same sum for thrice the time at half the rate?

97. Two casks of 48 and 42 gallons are filled with mixtures of spirits and water, the proportions being 13 : 7 and 18 : 17 in the two casks, respectively. If the contents of the two casks be mixed, and 20 gallons of water be added to the whole, what will be the proportion of spirits to water in the result?

98. There are four vessels of equal capacity. The first is filled with spirit to the extent of one-eighth, the second to one-sixth, the third to one-fourth, and the last to one-third. The first is then filled up with water, and from this mixture the second is filled up; again, from this second mixture the third is filled up, and in like manner the fourth from the third. What proportion of spirit to water is there in the fourth vessel?

99. A and B have each a certain amount of stock in the 3 per cents., making A's income 16*s.* more than B's. A sells out at 90, and invests the proceeds in $5\frac{1}{4}$ per cent. stock at 105; B sells out at 91, and invests his proceeds in 4 per cent. stock at 84. A's income is now £3 12*s.* more than B's. What amount of 3 per cent. stock did each possess at first?

100. A certain piece of work is to be performed by A, B, and C in 7 hours; and they can undertake it, although B cannot begin along with A and C. How long after them must he begin, if A could do the whole by himself in $15\frac{1}{6}$ hours, B the whole in $16\frac{1}{4}$ hours, C the whole in $18\frac{1}{5}$ hours?

ANSWERS.

Exercises 1

1. £7 2*s*. 3*d*.
 £6 8*s*. 4½*d*.
2. £24 12*s*. 6*d*.
3. 325 a. 2 r. 15 p.
 257 a. 3 r. 22 p.
4. $151\frac{13}{36}$; $83\frac{17}{36}$.
5. $\frac{83}{113}$; $\frac{15}{113}$.
6. 39½ mi.; 34½ mi.
7. $2\frac{4}{5}$; $2\frac{11}{30}$.
8. 824; 476 yds.
9. $\frac{107}{360}$; $\frac{23}{360}$.
10. A B 86, B C 93, A C 56.
11. £430.
12. 187 and 81 feet, *nearly*.

Exercises 2.

1. About 640 miles
2. 433.
3. $186\frac{7}{8}$ sq. ft.
4. $8\frac{4}{7}$ ft.
5. 353 yds.
6. 24·14 in.
7. 26·9258 ft.
8. 270·224 sq. in.
9. 72·16.
10. 31464; 171.
11. 160; 308.
12. 740.
13. 60, 91, 80.
14. 36·4, *nearly*.
15. 52, *nearly*.
16. 278·86 sq. in.

Exercises 3.

1. 297 lb.
2. 8 days 8 hrs.
3. $5\frac{4}{19}$ hrs.
4. 4 days.
5. 4*s*. 9*d*.; 3*s*. 2*d*.
6. 33*s*.; 18*s*.
7. £27 11*s*.
8. $4\frac{2}{3}$ days.
9. 8 : 3.
10. $\frac{5}{6}$; $\frac{1}{9}$.
11. 83, 67.
12. ·76, ·24.
13. £31 15*s*. 10*d*.
14. 16 men, 27 wom.
15. Cod 9*d*. a lb.
 Oysters 1*s*. 6*d*. a doz.
16. Beef, 6*s*. 3*d*.
 Mutton, 6*s*. 8*d*.

Exercises 4.

1. $3\frac{3}{7}$ days.
2. 14 hrs.
3. $7\frac{1}{5}$ days.
4. $95\frac{5}{53}$ m.
5. $10\frac{1}{10}$ min.
6. $3\frac{3}{5}$ hrs.
7. B 45 hrs, C $56\frac{1}{4}$,
 D $46\frac{7}{8}$; all, $12\frac{32}{39}$.
8. 13 cwt. 107 lb. $2\frac{5}{12}$ oz.
9. A $6\frac{7}{8}$ days, B $8\frac{1}{4}$.
10. $6\frac{1}{2}$ days.
11. 20 : 15 : 13.
12. 15 days.
13. $19\frac{3}{8}$ days.
14. 6 hrs. 40 min.
15. 8 days.
16. A 9 hrs., B $11\frac{1}{4}$, C 15.

Exercises 5.

1. A £160, B £300, C £135, D £210.
2. 1121·2 and 2838 sq. ft.
3. $3\frac{1}{3}$ miles.
4. 27 cwt. 48 lb., 22 cwt. 96 lb.
 14 „ 32 „ 21 „ 48 „
5. $\frac{53\frac{3}{4}}{75\frac{1}{4}}$.
6. 11 feet.
7. Base 33·86, perp. 19·35 in.
8. A 40, B 16.
9. 3 a. 2 r. 11 p. $7\frac{1}{4}$ yds.
10. £14240.
11. £184 3*s*. 4*d*.
 £171 15*s*. 5*d*.
 £178 12*s*. 10*d*.
12. 1800, 1600, 1440 sq. yds.
13. 16*s*. 6*d*.
14. £150 15*s*., £140 14*s*., £117 5*s*.
15. $62\frac{2}{9}$, $28\frac{4}{9}$, $53\frac{1}{3}$ sq. in.
16. £1200, £875.
17. 29 ft. 2 in.; 20 ft. 5 in.
18. 38·485 sq. ft.
19. £153 16*s*. $11\frac{1}{13}d$.
 £152 12*s*. $3\frac{9}{13}d$.
 £149 0*s*. $9\frac{3}{13}d$.

Exercises 6.

1. £1 7*s*., £1 10*s*., £4 10*s*.
2. £3 10*s*., £3 5*s*., £2 15*s*., &c.
3. $7\frac{1}{2}$, $5\frac{1}{4}$, $10\frac{3}{4}$ acres.
4. 30*s*., 20*s*., 16*s*., 54*s*.
5. 12, 18, 5, 45.
6. 3, $8\frac{1}{4}$, 1, 18, sq. yds.
7. £255, £235.
8. A 9*s*. 8*d*., B 16*s*. 8*d*., C 17*s*. 10*d*.
9. $\frac{19}{45}$.

Exercises 7.

1. 5 hrs. 20 min.
2. 86 yds.; 72 secs.
3. 61 hrs. 48 min.
4. 10 hrs. $5\frac{5}{11}$ m. & 10 h. $38\frac{2}{11}$ m.
 10 „ $54\frac{6}{11}$ „ ; 10 „ $21\frac{9}{11}$ „
5. 14 mi. $6\frac{1}{4}$ fur.
6. A 10 hrs.; B $6\frac{2}{3}$ hrs.
7. 24 min.
8. 23.
9. 1 hr. $25\frac{1}{2}$ min.; $42\frac{3}{4}$ min.
10. B first; 4 ft. $5\frac{3}{4}$ in.
11. B wins by 88 yds.
12. 21 miles.
13. $7\frac{1}{2}$ mi.; 42 mi.

Exercises 8.

1. 12 per cent.
2. 1*s*. $9\frac{3}{4}d$.
3. 3 cwt. 79 lb.
4. 11*s*. 8*d*.
5. 26*s*. 10*d*.
7. $31\frac{1}{4}$ per cent.
8. $13\frac{1}{23}$ per cent.
9. 30.
10. £500.
11. 18*s*.
12. 90*s*.
13. $11\frac{1}{9}$ per cent.
14. 33*s*. $0\frac{5}{9}d$.
15. 2 months.

Exercises 9.

1. £3365.
2. £1 17*s.* 6d. more.
3. £9876$\frac{44}{81}$.
4. £1 8*s.* 6*d.* more.
5. 5 per cent.
6. £3690.
7. $4\frac{1}{4}$ per cent.
8. £2890 10*s.*
9. £9 15*s.* 4*d.* more.
10. 18*s.* 9*d.*
11. $4\frac{1}{4}$ p. cent. stock.
12. £19$\frac{5}{6}$ less.
13. £25000.
14. 83$\frac{3}{4}$.
15. £10 9*s.* 9$\frac{3}{4}$*d.*
16. 94.

Exercises 10.

1. £4 15*s.* 4*d.*
2. 120 yards.
3. £6 4*s.* 7*d.*
4. 4*d.*
5. 15 in.
6. 27 ft. long, 18 wide, and 14$\frac{1}{2}$ high.
7. 16 ft.
8. 13$\frac{1}{2}$ ft.
9. £8 6*s.* 8*d.*

Exercises 11.

1. 61 miles.
2. 63 „
3. 29 „
4. 38 „
5. 52 „
6. 67 „
7. 51 miles.
8. 45 „
9. 14 „
10. 65 „
11. 23° S.; 952 mi.
12. 34° S.; 851 mi.
13. 48° N.; 690 mi.
14. 31° N.; 885 mi.
15. 38° S.; 817 mi.
16. 58° N.; 537 mi.
17. 21° N.
18. 31° N. and S.

Exercises 12.

Miles.	Miles.	Miles.	Miles.
1. 1210.	5. 620.	9. 365.	13. 99.
2. 1088.	6. 5120.	10. 1380.	14. 3780.
3. 550.	7. 2830.	11. 6660.	
4. 115.	8. 5560.	12. 710.	

Exercises 13.

1. 7 h. 4 m. A.M.
 2 1 P.M.
 1 56 P.M.
2. 8 0 A.M.
3. 9 h. 54 m. P.M.
4. 11 5 A.M.
5. 4 41 A.M.
6. 2 18 P.M.
7. 8 h. 41 m. A.M.
8. 22° 30′ E.
9. Long. 14° 42′ E.
10. 30° E.

Exercises 14.

1. $21\frac{1}{2}$, $38\frac{1}{2}$.
2. 240.
3. $11\frac{4}{11}$ per cent.
4. 4 o'clock P.M.
5. $4\frac{4}{11}$ days.
6. 2*s.* 8*d.*
7. 6 hrs. 48 min.
8. 56 doz.
9. 9 florins.
10. 4 hrs. 57 min.
11. 11.
12. A £$21\frac{1}{2}$, B £$65\frac{1}{2}$, C £$35\frac{1}{2}$.
13. $11\frac{1}{4}$, $88\frac{3}{4}$.
14. 12 pears.
15. 1 o'clock A.M.
16. 450 men, 400 wom., 150 chil.
17. $8\frac{1}{6}$ days.
18. $\frac{49}{592880}$.
19. $21\frac{7}{11}$ min. and $43\frac{7}{11}$ min. past 6.
20. $31\frac{1}{2}$ hours.
21. £10 2*s.*
22. 6 hrs. 38 min. A.M.
23. 16 m., 24 w., 18 b.
24. 27·4 miles.
25. £750.
26. 5 mi. and $3\frac{1}{2}$ mi.
27. A 800 acres, B 500, C 300.
28. 23, 17.
29. 36 miles.
30. $9\frac{1}{7}$ miles.
31. $25\frac{1}{7}$ min. and $51\frac{3}{4}$ min. past 7.
32. $124\frac{1}{2}$, $191\frac{1}{2}$ yds.
33. £1829 6*s.* 8*d.*
34. $7\frac{119}{334}$ days.
35. £$273\frac{1}{3}$, £375.
36. £625.
37. 630 sq. in.
38. 54, $22\frac{1}{2}$, $20\frac{1}{4}$.
39. $7\frac{1}{9}$ days.
40. 2·45 in. per min.
41. $22\frac{7}{44}$ mi. an hour.
42. 6° 25′ W.
43. 12 per cent.
44. B 45 hrs., C $56\frac{1}{4}$, D $46\frac{7}{8}$; all $12\frac{32}{39}$.
45. 42 at 3 for $2\frac{1}{2}d.$, 48 at 8 for $6\frac{1}{2}d.$
46. 10 men, 16 wom. 20 boys.
47. £$94\frac{1}{2}$, £168, £396.
48. $5\frac{3}{5}$ days.
49. 12 min. $31\frac{1}{11}$ sec.
50. 5 men.
51. $4\frac{1}{4}$ lb. T, $9\frac{3}{4}$ lb. C.
52. 1170 miles.
53. 51 dozen.
54. 11 : 12.
55. £1750.
56. 6 hrs. 52 min.
57. £81 12*s.*; $9\frac{1}{2}$ mo.
58. 12 men, 10 wom.
59. $11\frac{71}{308}$ sec.
60. A 12 days, B 15, C 20.
61. 6 years.
62. $91\frac{1}{4}$.
63. 9 months.
64. 28*s.* 4*d.*, 41*s.* 8*d.*
65. 8 months.
66. $13\frac{1}{11}$, $44\frac{1}{11}$, $2\frac{1}{11}$, and $476\frac{8}{11}$.
67. £364, £$227\frac{1}{2}$, £245.
68. £1666 13*s.* 4*d.*
69. $20\frac{1}{2}$ pints.
70. £1800.
71. 239 feet.
72. C £15, S 37*s.* 6*d.*
73. £190, £152, £$142\frac{1}{2}$.
74. 818 yards.
75. $23\frac{9}{13}$, $122\frac{9}{13}$, $2\frac{5}{13}$, $1731\frac{3}{13}$.
76. £855, £805.
77. £6 19*s.* $7\frac{1}{7}d.$
78. 150 lb. S., 130 C.
79. 8*s.* $10\frac{2}{3}d.$
80. 6 hrs. 18 min.
81. 21*s.* $10\frac{18}{61}d.$ per ton.
82. £3420, £1680.
83. £$1793\frac{3}{4}$, £$1691\frac{1}{4}$, £1750.
84. 9, $6\frac{1}{3}$, 3 ft.
85. 2·08 miles.
86. $1\frac{1}{2}$ hr.
87. 24 gall.
88. £559 13*s.* 9*d.*
89. A gives 4 months
B „ 7 „
90. £720, £612, £680.
91. $6\frac{6}{7}$ hours.
92. 2 gall. from S, 14 „ „ T.
93. £1460.
94. $\frac{47}{49}$.
95. £11 12*s.* $2\frac{2}{5}d.$
96. £4 12*s.*
97. 12 : 13.
98. 61 : 35.
99. A £$1466\frac{2}{3}$, B £1440.
100. $4\frac{1}{2}$ hours.

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